SHIP MOTION ESTIMATION

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by

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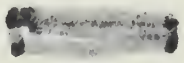
ABSTRACT

This thesis proposes a method of identifying the dynamics of ship angular motion at sea as the basis for ship motion estimation. Various sources of information are discussed. A digital model to simulate ship's motion is developed. Identification algorithms are presented with the results of their stochastic digital simulation. Sensitivity of identification error to sample rate was investigated. A plan for shipboard implementation utilizing a digital computer is presented.



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INTRODUCTION

The sophistication of modern shipboard systems is imposing accuracy requirements in measuring a ship's attitude. A 10 mil error in yaw correction to a gun platform is a 200 yard error at 10 nautical miles. System time delays and additive measurement noise are key sources of system error. Automatic carrier landing systems require that the predicted touch down point on the deck be known. Present system effectiveness degenerates rapidly as sea states increase. When the wave-off decision point is reached by the aircraft, the automated landing system must know the aircraft position relative to the desired impact point at the time of touchdown. If pitch and yaw can accurately be predicted, a true all weather capability may be anticipated. For these and many other applications the accurate knowledge of present position and the prediction of future positions are key elements of the system environment.

An accurate prediction of ship motion must necessarily involve information from the forcing function. Proper interpretation should lead to an identification of the current dynamic structure of the ocean waves. This could yield a more accurate input to the prediction of sea states and ocean weather propagation.

The French have done considerable work in the field of spectro-angular ocean wave analysis [1, 2] . However their work is successful only in predicting mean and extreme wave amplitudes and periods. The primary available inputs are local surface wind conditions distributed over the ocean area. Work in this area is not yet sufficiently advanced to be applied to a real time estimation problem.

The investigation of the statistical properties of ocean waves has been a well traveled route. R. L. Wiegel [3] from the University of California has done extensive work using both surface and bottom moored pressure sensing units. Data reduction again shows normal distributions of wave period and amplitude in open waters.

Ship motion estimation may be approached as a stochastic control problem. Thus the ship is the plant and the ocean wave is the forcing function with normal Gaussian distribution. Pitch, roll, and yaw will be treated as separate, uncoupled subsystems discretely sampled to facilitate digital simulation and data processing. Only the roll mode will be referred to in the remainder of this paper on the assumption that the other modes may be estimated by the same techniques.

PLANT SIMULATION

To evaluate a technique for ship motion estimation, it is first necessary to have a source of data. A fourth order plant composed of two cascaded identical second order systems was selected for motion simulation. This plant has known natural frequency (ω_n) and damping coefficient (ζ),

$$G(s) = \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right) \left(\frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \right) \qquad (1)$$

The plant may be represented by

$$\dot{X} = AX + BW \quad \text{where} \qquad (2)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_n^2 & -4\zeta\omega_n^3 & -4\omega_n^2(\zeta^2+.5) & -4\zeta\omega_n \end{bmatrix} \text{and } B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Selection of ζ and ω_n determines the dominant characteristics. The discrete recursive state equation in matrix form is

$$X(k+1) = \phi X(k) + \Gamma W(k). \qquad (3)$$

W(k) is the discrete, white Gaussian excitation with zero mean. The desired system output is the discrete time sequence of roll angle corresponding to X_1 where

$$X_1(k) = HX(k), \quad H = (1\ 0\ 0\ \ldots).$$

Additive measurement noise, v(k), may be included at the output to complete the simulation model, fig. 1. V(k) is assumed to be white Gaussian noise, uncorrelated with W(k). The process is described by

$$z(k) = HX(k) + v(k). \qquad (4)$$

It will be convenient to transform ϕ into the canonic companion matrix form, ϕ^*, using Lee's observability criteria [4], giving

$$D = \begin{bmatrix} H \\ H\phi \\ \vdots \\ H\phi^{n-1} \end{bmatrix},$$

$$\phi^* = D\,\phi\,D^{-1} = \left[\begin{array}{c|c} 0 & I \\ \hline a_1\ a_2 \cdots a_n \end{array}\right], \tag{5}$$

and the companion distribution matrix

$$\Gamma^* = D\,\Gamma. \tag{6}$$

From the work of Lee and Ho [5] this defines a new recursion equation with the same output values as equation (4) as seen below

$$\begin{aligned} Y(k+1) &= \phi^* Y(k) + \Gamma^* W(k) \\ \mathbf{z}(k) &= H\,Y(k) + v(k) \end{aligned} \tag{7}$$

where W(k), H, v(k), and $\mathbf{z}(k)$ are the same but the Y elements are different.

Representing the simulation model in this form reduces both the number of computations and the time required to generate a sample set of data on a digital computer. Since the data set is unchanged by the foregoing manipulations, the dominant characteristics are preserved. Values of $\zeta = 0.7$ and $\omega_n = 0.2\,\pi$ were used as reasonable estimates for the roll dynamics of a moderate size ship. The excitation, W(k), was a random number generating subroutine with zero mean and standard deviation of one. Multiplying the random numbers by a constant permitted simulation of any desired sea state. An example of roll angles generated using a constant of twenty and a sample period of 0.1 second is shown in fig. 2.

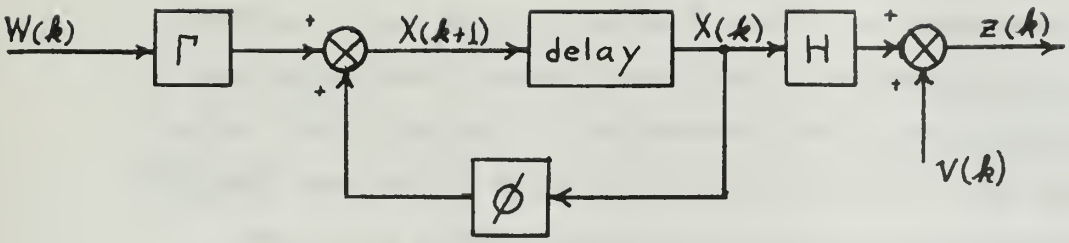


Fig. 1. Block Diagram of the Simulation Model.

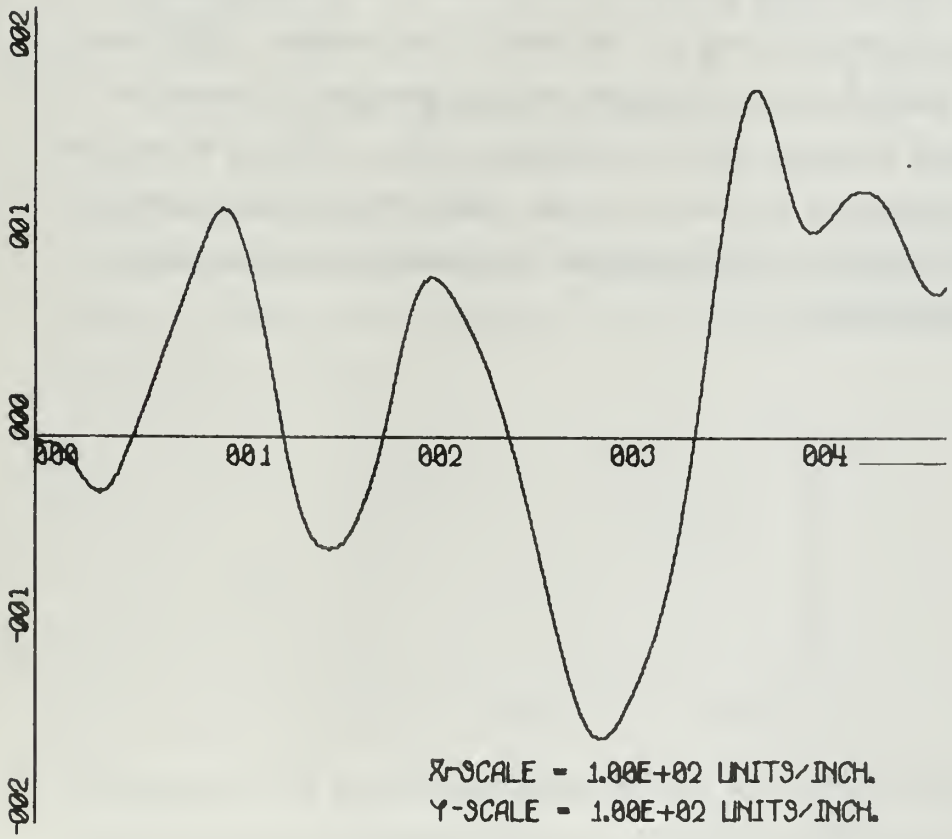


Fig. 2. Roll Angles vs Time (10 sec/in) Generated by the Simulation Model Excited with Discrete Gaussian Noise, $\sigma = 20$, at the Sampling Interval, 0.1 sec. Dominant Period = 10 sec, $\gamma = 0.7$.

IDENTIFICATION

The problem of identification arises when the set of differential equations describing the system dynamics are not known, uncertain, or are subject to periodic or continuous change. On a large time scale the ship dynamics may be expected to change as fuel, stores, ammunition, etc. vary in quantity and location. Short time changes may be anticipated due to changes in ships heading and speed.

In state vector notation the recursion equation for ship roll motion (uncoupled) will have the form;

$$X(k+1) = \Phi X(k) + \Gamma W(k) \tag{8}$$

where W(k) is the discrete Gaussian excitation. Φ is the fixed state transition matrix of rank N, the order of the system. The term fixed is used here on the assumption that the changes in dynamics mentioned above occur at such a sufficiently slow rate that Φ may be adequately represented as time invariant over a finite time interval. Thus Φ must be periodically identified to follow ship motion within accuracy specifications.

IDENTIFICATION OF THE FREE DYNAMIC SYSTEM

A free dynamic system may be represented by the equation

$$Y(k+1) = \phi * Y(k) \ . \tag{9}$$

Lee developed an identification scheme for a plant having no numerator dynamics [4]. Lee's development gave the following equations;

$$\hat{\phi} * = S_{2n}(S_{2n-1})^{-1} \tag{10}$$

where

$$S_{2n-1} = \begin{bmatrix} z_1 & z_2 & \cdots & z_n \\ z_2 & z_3 & & \cdot \\ \cdot & \cdot & & \cdot \\ z_n & z_{n+1} & \cdots & z_{2n-1} \end{bmatrix}, \quad S_{2n} = \begin{bmatrix} z_2 & z_3 & \cdots & z_{n+1} \\ z_3 & z_4 & & \cdot \\ \cdot & \cdot & & \cdot \\ z_{n+1} & \cdots\cdots & & z_{2n} \end{bmatrix}$$

and $z(k) = HY(k)$, a scalar.

Using the same data, the order of the system may also be identified. Choose some M greater than N and build an M x L matrix, A. $L = 1,2,3,\ldots N,N+1,\ldots$.

$$A = \begin{bmatrix} z_1 & z_2 & \cdots\cdots & z_{\ell} \\ z_2 & z_3 & \cdots\cdots & z_{\ell+1} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ z_m & z_{m+1} & \cdots\cdots & z_{m+\ell-1} \end{bmatrix}$$

The product $A^T A$ will be positive definite for L less than or equal to N and singular for L greater than N. Hence the order of the system is the maximum nonsingular value of L.

A problem arises from the possible occurrence of numerator dynamics which leads to residues (errors) in the identification. In z-transform theory the N^{th} row of ϕ * corresponds to the denominator coefficients of the system z-transform. The numerator remains

unidentified. Hence, if there is an unknown non-scalar numerator in the real plant corresponding to a nonzero initial condition, errors will result.

Identification using this method was investigated with excellent results by setting an initial condition for roll angle and then releasing the plant. If excitation is applied while measurements are being taken, the plant is no longer a free dynamic system. Identification attempted with random excitation applied was inaccurate and unreliable.

IDENTIFICATION FOR THE STOCHASTIC CASE

Using the transformed recursion equation in the new state space, Lt. Ralph Hudson's unpublished doctoral notes show the development of a straight forward identification scheme as follows:

$$Y(k+1) = \phi * Y(k) + \omega(k) \tag{11}$$

where

$$Y(k) = \begin{bmatrix} z_{k-n+1} \\ \cdot \\ \cdot \\ z_{k-1} \\ z_k \end{bmatrix}, \quad \omega(k) = \Gamma * W(k),$$

and the scalar, W(k), represents the white Gaussian excitation. If equation (11) is post multiplied by $Y(k)^T$;

$$Y(k+1)Y(k)^T = \phi * Y(k)Y(k)^T + \omega(k)Y(k)^T \tag{12}$$

Taking the expectation of both sides;

$$E[\,Y(k+1)Y(k)^T] = \phi * E[\,Y(k)Y(k)^T] + E[\omega(k)Y(k)^T] \tag{13}$$

Notice that the $E[\,Y(k+1)Y(k)^T]$ is just the autocorrelation function, $R(\tau)$, for tau equal to one. Similarly, $E[\,Y(k)Y(k)^T]$ is the autocorrelation for tau equal to zero. Therefore $\hat{\phi} * = R(1)R(0)^{-1}$.

If the order of the identification is M and $M \leq N$, the true system order, the inverse will exist. This is implied by the fact that an N^{th} order linear system may be defined by N linear independent differential equations. Hence any $M \leq N$ of these equations are also independent. If $M > N$ independence is lost and R(0) of rank $M > N$ is singular. Thus if N is unknown, it may be identified by testing for the largest non-singular rank of R(0).

For application to discrete linear (Kalman) filtering it is also useful to note that the Q required for determining the gain matrix may be concurrently identified as follows;

$$Q = \Gamma * E[\,W(k)W(k)^T]\Gamma *^T = E\,[\,\omega(k)\,\omega(k)^T] \tag{14}$$

Hence

$$Q = E\left[\left\{Y(k+1) - \phi^* Y(k)\right\}\left\{Y(k+1) - \phi^* Y(k)\right\}^T\right]$$

which simplifies to

$$Q = R(0) - \phi^* R(1)^T. \tag{15}$$

This scheme was programmed in Fortran 63 for simulation on the CDC 1604 digital computer using data from the simulation model. Batch processing and recursive identification were investigated.

1. Batch processing: A sample set of roll angles was generated and stored. The stored data was then batch processed to form the auto-correlation functions, R(0) and R(1), for the set. A matrix inversion routine was used to invert R(0). ϕ^* was then identified as the matrix product, $R(1)R(0)^{-1}$.

If Γ^* was restricted to a single non zero element, identification of the fourth order plant was good using 50 samples and excellent for 500 samples. However, if Γ^* had all non zero elements, residues due to numerator dynamics caused errors as great as 110% of the true values of the elements. Increasing the sample size from 500 to 900 showed no improvement.

2. Recursive identification: Hudson's identification scheme may be manipulated into the following recursion equations by using a matrix inversion lemma.

$$\hat{\phi}^*(k+1) = \hat{\phi}^*(k) + \left(Y(k)^T P(k) Y(k) + 1\right)^{-1}\left(Y(k+1) - \hat{\phi}^*(k) Y(k)\right) Y(k)^T P(k)$$

$$P(k+1) = P(k)\left(I - \left[Y(k)^T P(k) Y(k) + 1\right]^{-1} Y(k) Y(k)^T P(k)\right)$$

where $P(k) \equiv R(0)^{-1}$.

Let $\beta(k) = \left(Y(k)^T P(k) Y(k) + 1\right)^{-1}$, a scalar.

Then

$$\hat{\phi}^*(k+1) = \hat{\phi}^*(k) + \beta(k)\left(Y(k+1) - \hat{\phi}^*(k) Y(k)\right) Y(k)^T P(k) \tag{16}$$

and

$$P(k+1) = P(k)\left(I - \beta(k)Y(k)Y(k)^{T}P(k)\right) . \qquad (17)$$

The above relationships identify an N by N ϕ * matrix of N^2 elements. For the scalar observable case N(N-1) of these elements are already known to be zeros or ones. Hence N^2 elements are identified to learn N unknowns in the N^{th} row. Thus a more efficient method of the foregoing has been derived by Yu-Chi Ho [5] and R.C.K. Lee [4,5] which recursively estimates the elements of the N^{th} row.

$$\hat{\phi}(k+1) = \hat{\phi}(k) + P(k)Y(k)\left(z(k+1) - \hat{\phi}(k)^{T}Y(k)\right)\beta(k) \qquad (18)$$

$$P(k) = P(k+1)\left(I - \beta(k-1)Y(k-1)Y(k-1)^{T}P(k-1)\right) \qquad (19)$$

where z(k+1) = the scalar measurement at stage (k+1)

$\hat{\phi}(k)$ = (Nx1) column such that $\hat{\phi}(k)^{T}$ is equal to the N^{th} row of ϕ *(k)

Initialization of the recursive equations might be accomplished for $\hat{\phi}^{*}(0) = S_{2n}S_{2n-1}^{-1}$ or by using a small sample set of 2N+1 or more measurements and forming R(1) and $R(0)^{-1}$ where by $\hat{\phi}^{*}(0) = R(1)R(0)^{-1}$ and $P(0) = R(0)^{-1}$. Both of these methods require the complicated matrix inversion routine. From Lee's work [4] the convergence rate appears to be satisfactory for any reasonable estimate of $\hat{\phi}^{*}(0)$ or $\hat{\phi}(0)$. Experience seems to bear this out for the simulation model. The P matrix ought to be inversely proportional to time approaching zero as $\hat{\phi}$ approaches ϕ . P has its most significant effect on the initial rate of convergence. Experience indicates that if P is initialized with a reasonably large set of values on the main diagonal ($\sim 10^{6}$), convergence is satisfactory. Additional work in this area would be useful to define an optimal P(0), perhaps in terms of initial convergence rate, measurement noise statistics, and identification error as a function of time.

Convergence tests were run using the scalar observable roll angles generated by the fourth order model. A ten second period, damping

coefficient of 0.7 and an excitation constant of twenty were used. The sample period was one second. Three cases were investigated using the same sample set.

The first case was with Γ^* suppressed to zeros in all but the last element to eliminate numerator dynamics. The convergence of 600 samples is shown in fig. 3a,b,c,d for each of the fourth row elements of $\hat{\phi}^*$. The second case was the same as the first but with Gaussian additive measurement noise having a standard deviation of one. Excellent convergence is shown in fig. 4a,b,c,d. The third case used the entire Γ^* vector and no measurement noise. The residues are apparent in fig. 5a,b,c,d. Comparing the eigenvalues of ϕ^* with those of $\hat{\phi}^*$ with residues indicates a considerable shift, fig. 6. The responses to different sample rates in the following section indicates that the residues may be minimized sufficiently to provide acceptable root locations.

The true computed values for the plant with a 10 sec period sampled at 1 sec intervals are as follows,

$$\phi^* = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -.1722 & .9633 & -2.177 & 2.322 \end{bmatrix}$$

$$\Gamma^* = \begin{bmatrix} .105 \\ .245 \\ .500 \\ .877 \end{bmatrix}, \qquad \Gamma^* \text{ suppressed} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ .877 \end{bmatrix}.$$

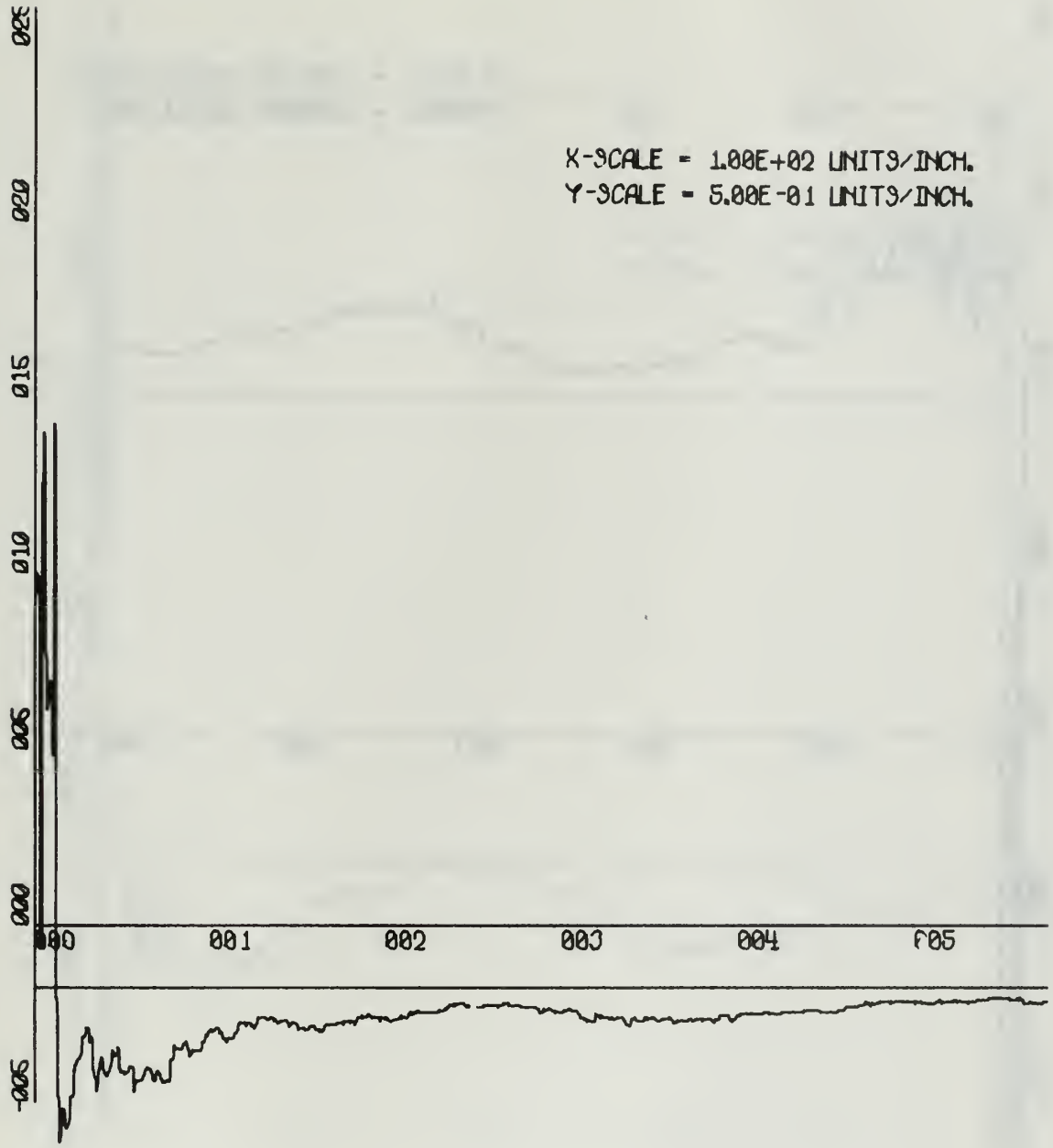
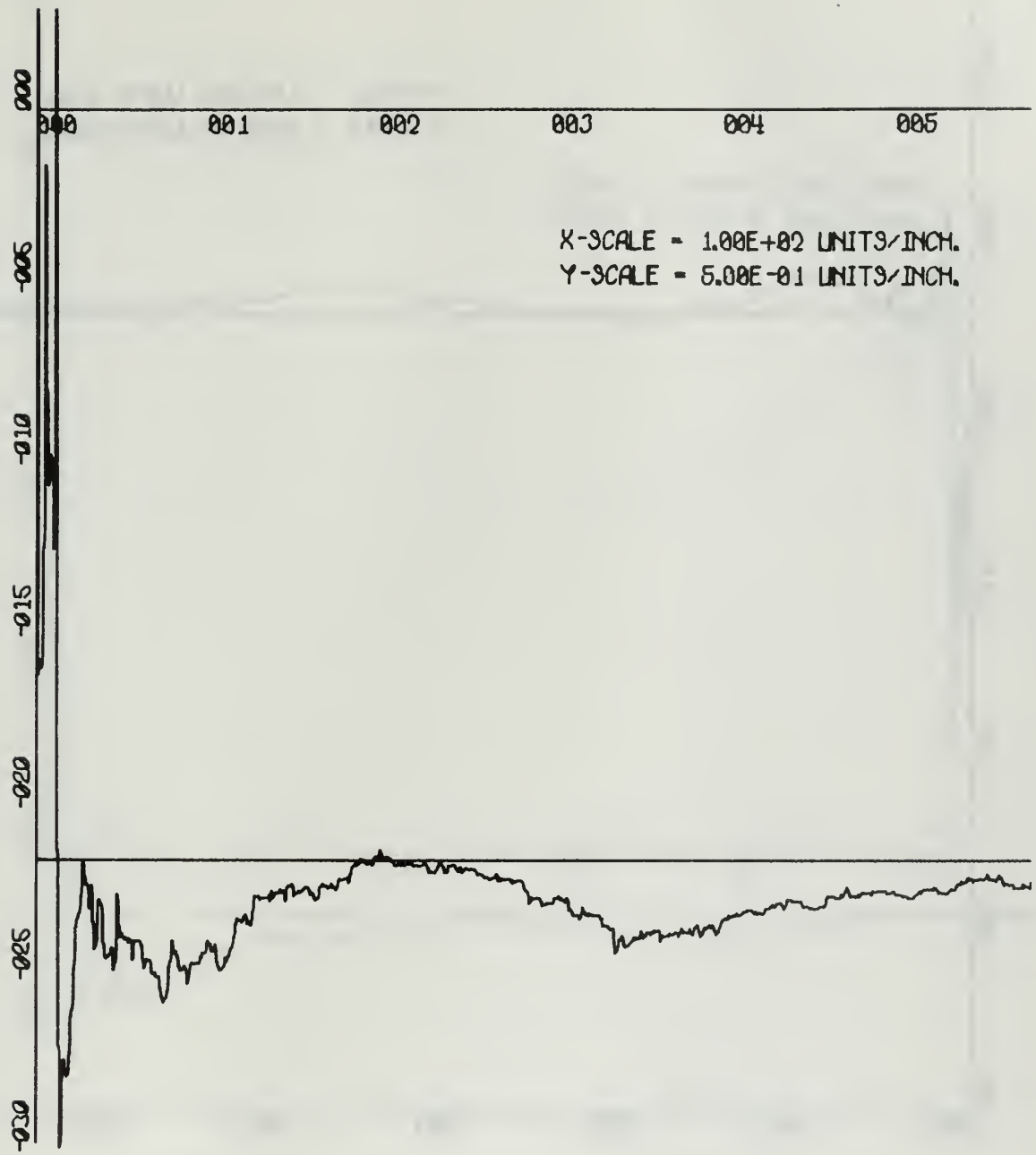


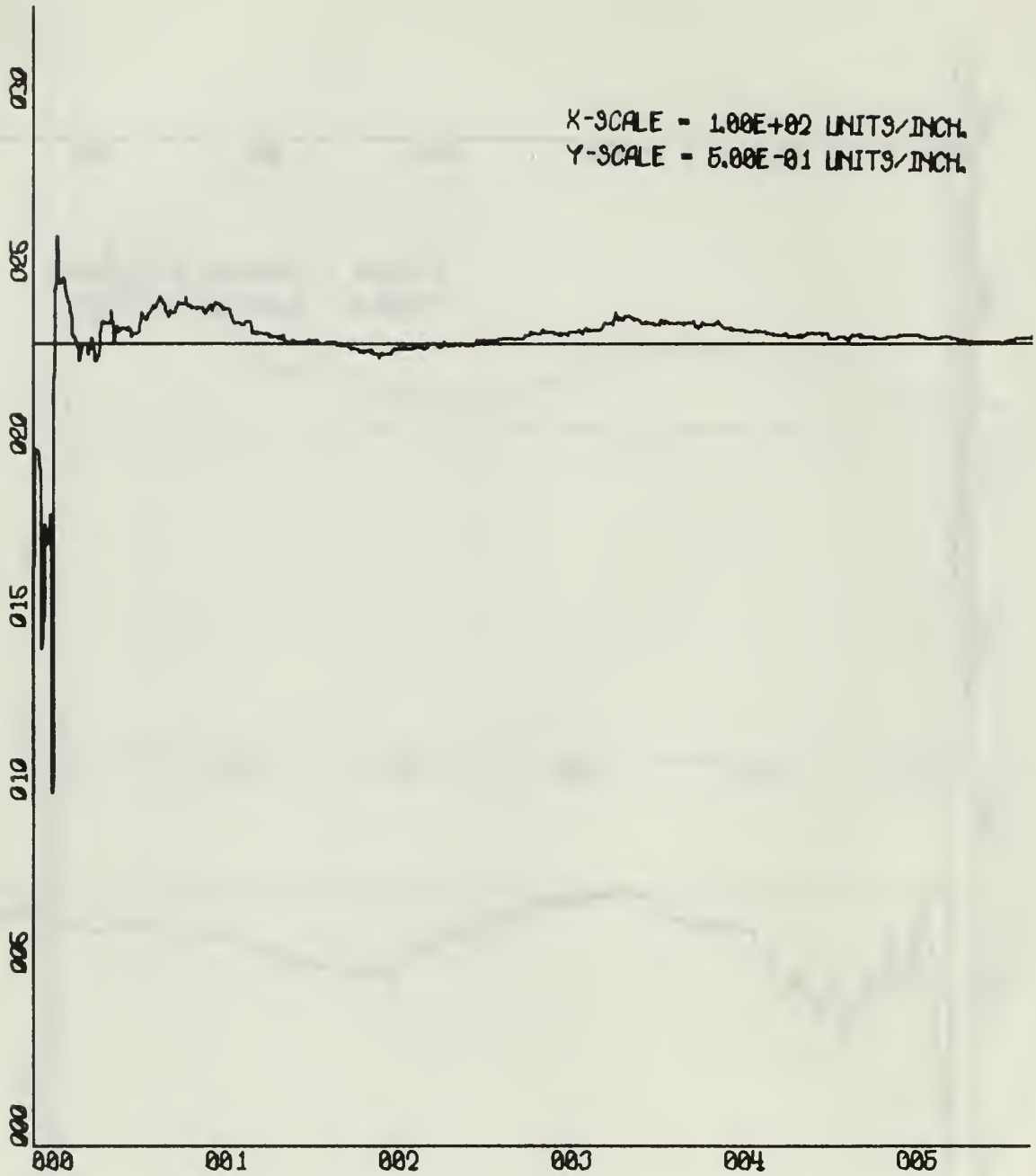
Fig. 3. Recursive Convergence of $\hat{\phi}^*$ vs Sample Number, No Measurement Noise, 10 sec Dominant Period Plant with Zero Initial Conditions. Discrete Gaussian Excitation, $\sigma = 20$, Applied at the Sample Interval of 1 sec. Γ^* Suppressed ($\Gamma^*(1,2,3) = 0$), $\gamma = 0.7$. The Straight Line Represents the True Value of the Particular ϕ^* Element. (a) $\hat{\phi}^*(4,1)$ vs Sample Number. (b) $\hat{\phi}^*(4,2)$ vs Sample Number. (c) $\hat{\phi}^*(4,3)$ vs Sample Number. (d) $\hat{\phi}^*(4,4)$ vs Sample Number.

(a) $\hat{\phi}^*(4,1)$ vs Sample Number, ($\phi^*(4,1) = -0.17217$)

(b) $\hat{\phi}^*(4,2)$ vs Sample Number, ($\phi^*(4,2) = 0.96328$)



(c) $\hat{\phi}^*(4,3)$ vs Sample Number, ($\phi^*(4,3) = -2.1772$)



(d) $\hat{\phi}^*(4,4)$ vs Sample Number, ($\phi^*(4,4) = 2.3215$)

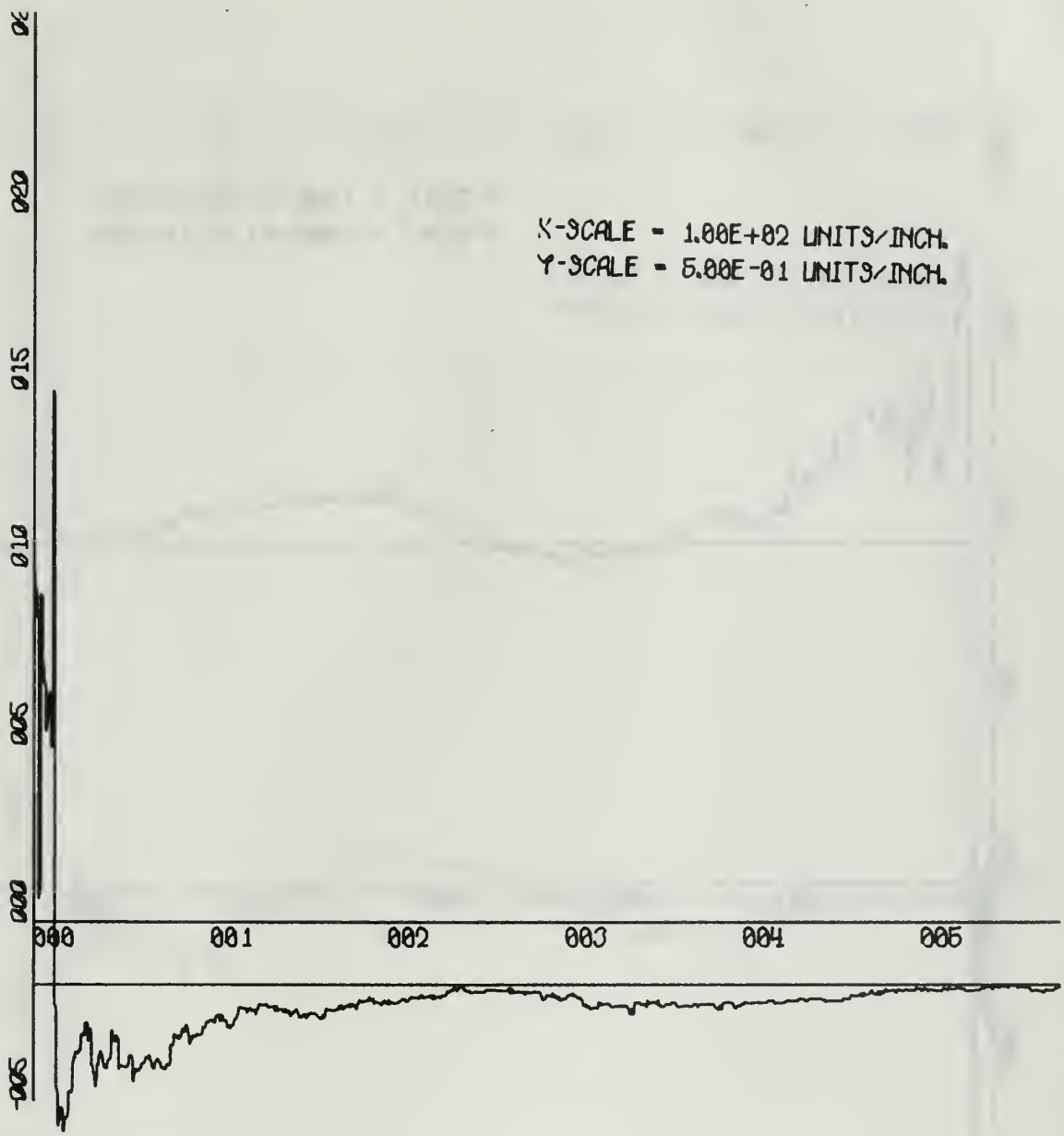
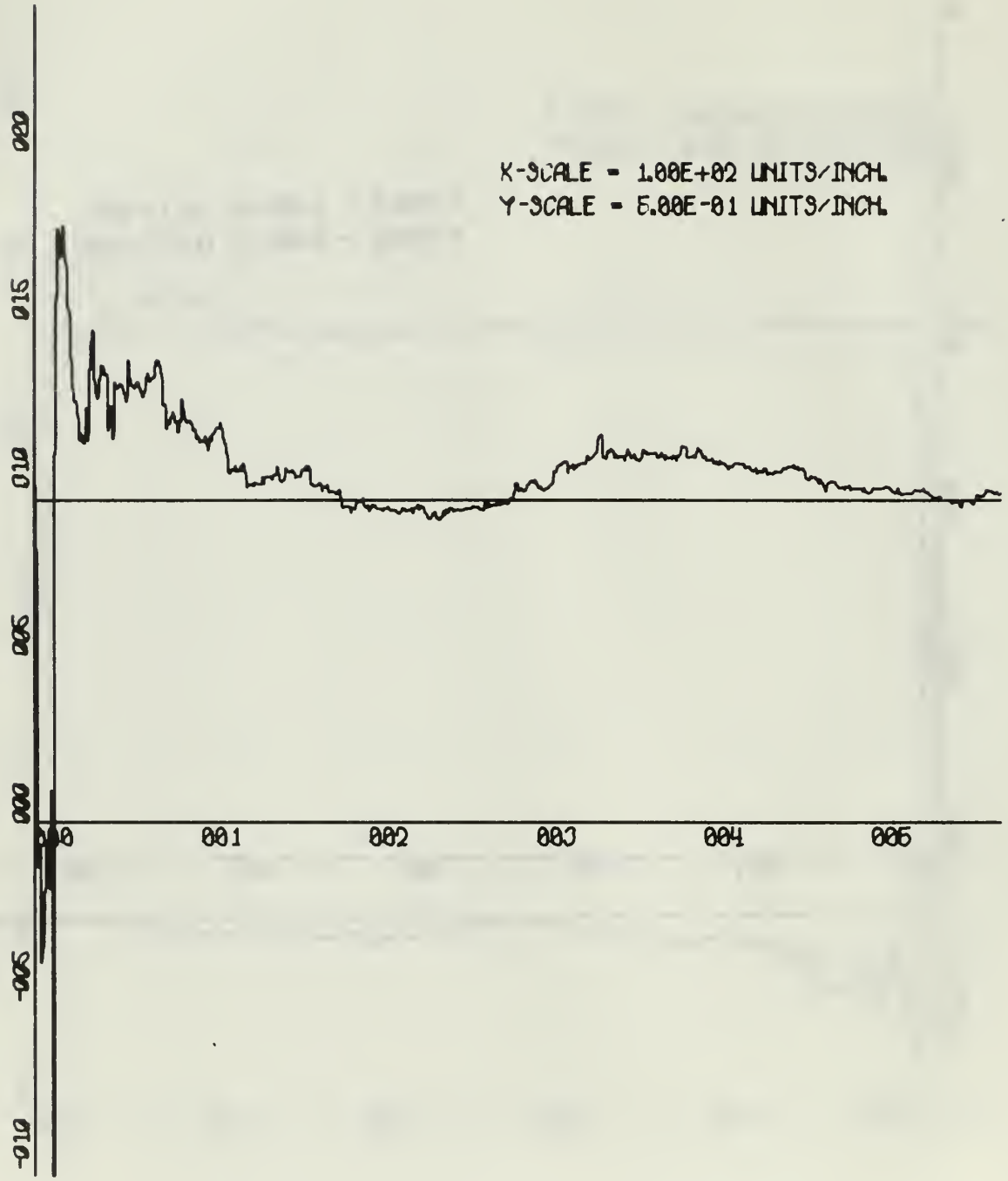
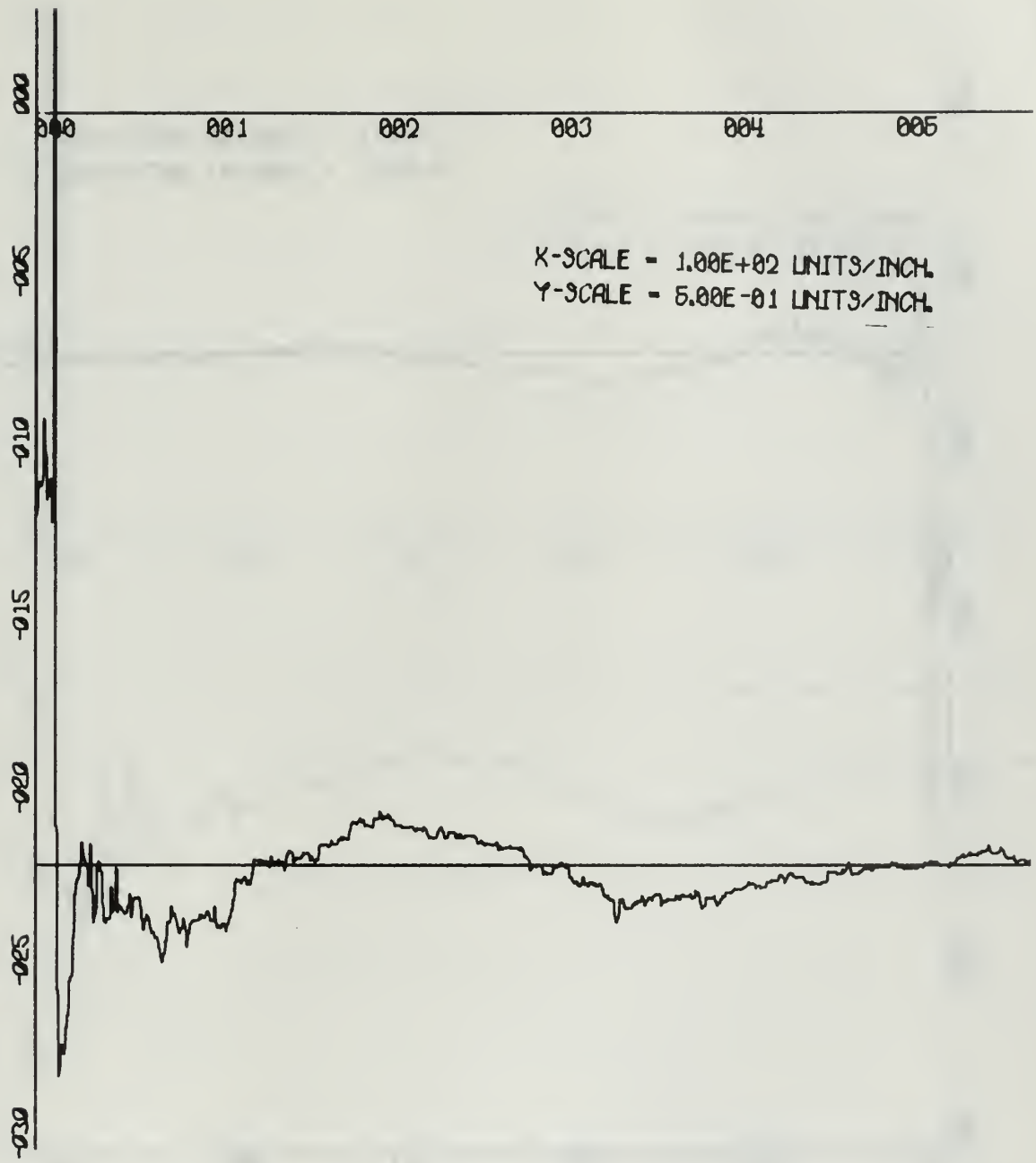


Fig. 4. Recursive Convergence of $\hat{\phi}^*$ vs Sample Number with Additive Gaussian Measurement Noise, $\sigma = 1$, using the 10 sec Dominant Period Plant, Zero Initial Conditions. Sampled and Excited at 1 sec Intervals, Γ^* Suppressed ($\Gamma^*1,2,3 = 0$), $\zeta = 0.7$. The Straight Line Represents the True Value of the Particular ϕ^* Element. (a) $\hat{\phi}^*(4,1)$ vs Sample Number. (b) $\hat{\phi}^*(4,2)$ vs Sample Number. (c) $\hat{\phi}^*(4,3)$ vs Sample Number. (d) $\hat{\phi}^*(4,4)$ vs Sample Number.

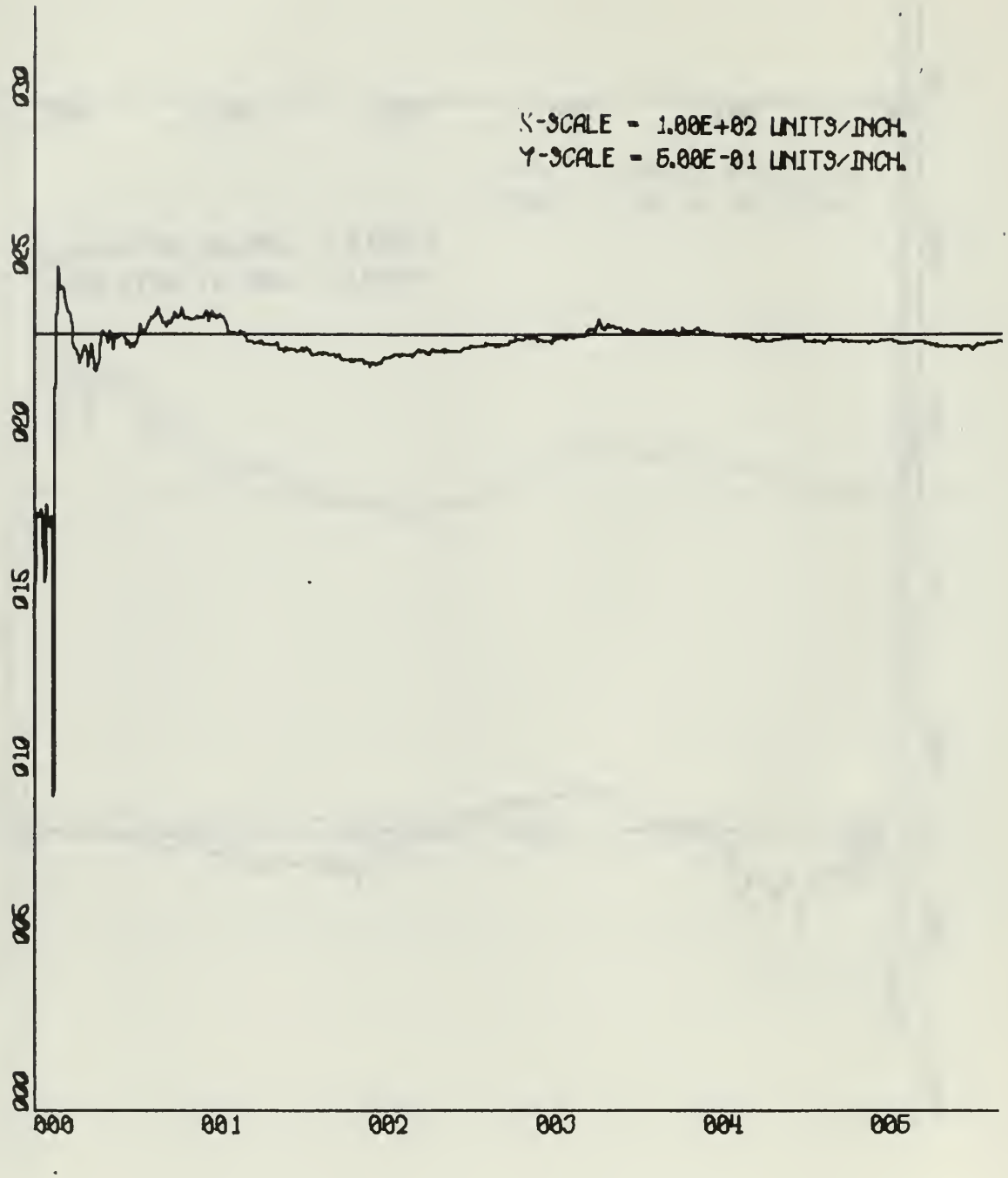
(a) $\hat{\phi}^*(4,1)$ vs Sample Number, ($\phi^*(4,1) = -0.17217$)



(b) $\hat{\phi}$ *(4,2) vs Sample Number, (ϕ *(4,2) = 0.96328)



(c) $\hat{\phi}$*(4,3) vs Sample Number, (ϕ*(4,3) = -2.1772)



(d) $\hat{\phi}$*(4,4) vs Sample Number, (ϕ*(4,4) = 2.3215)

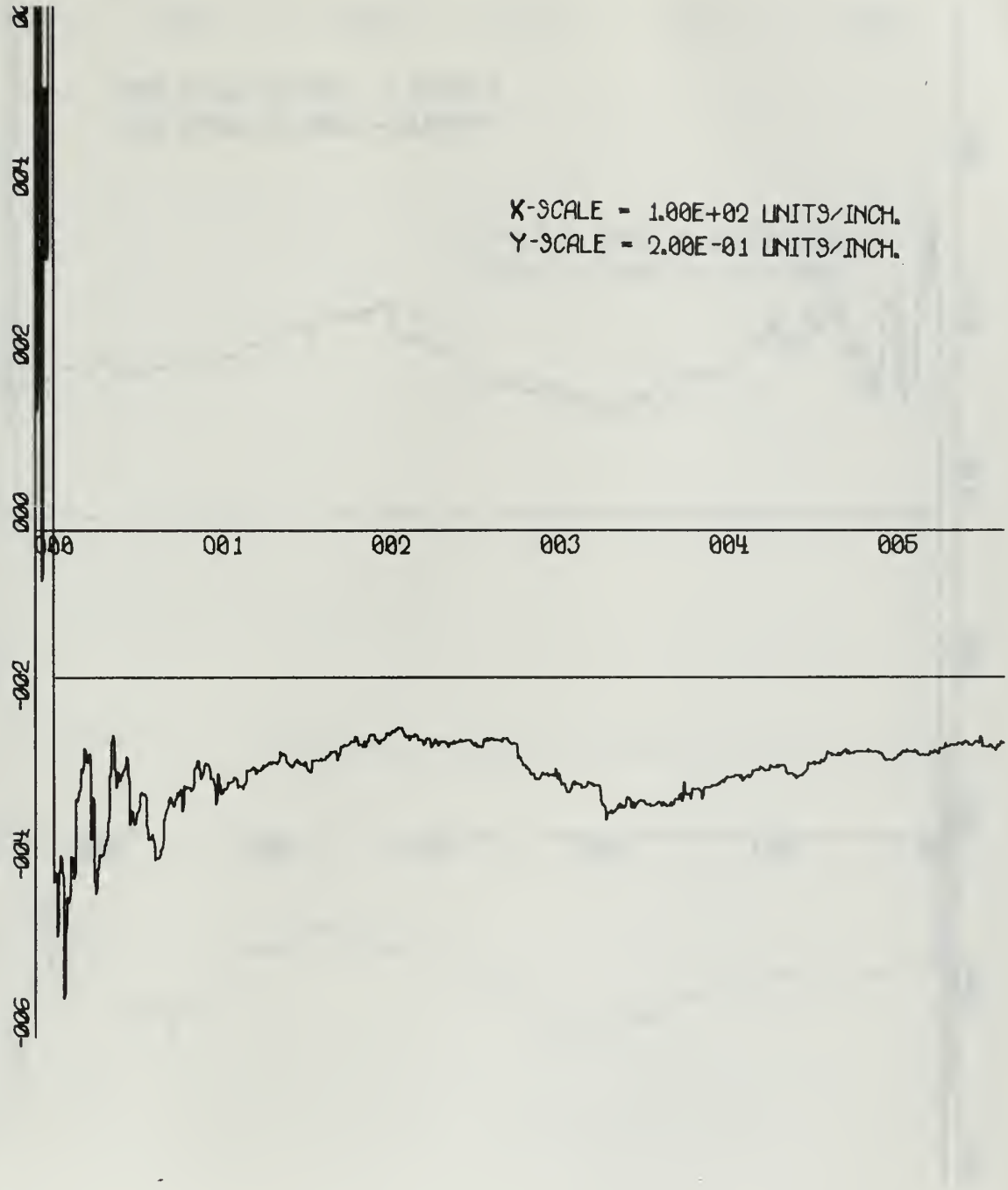
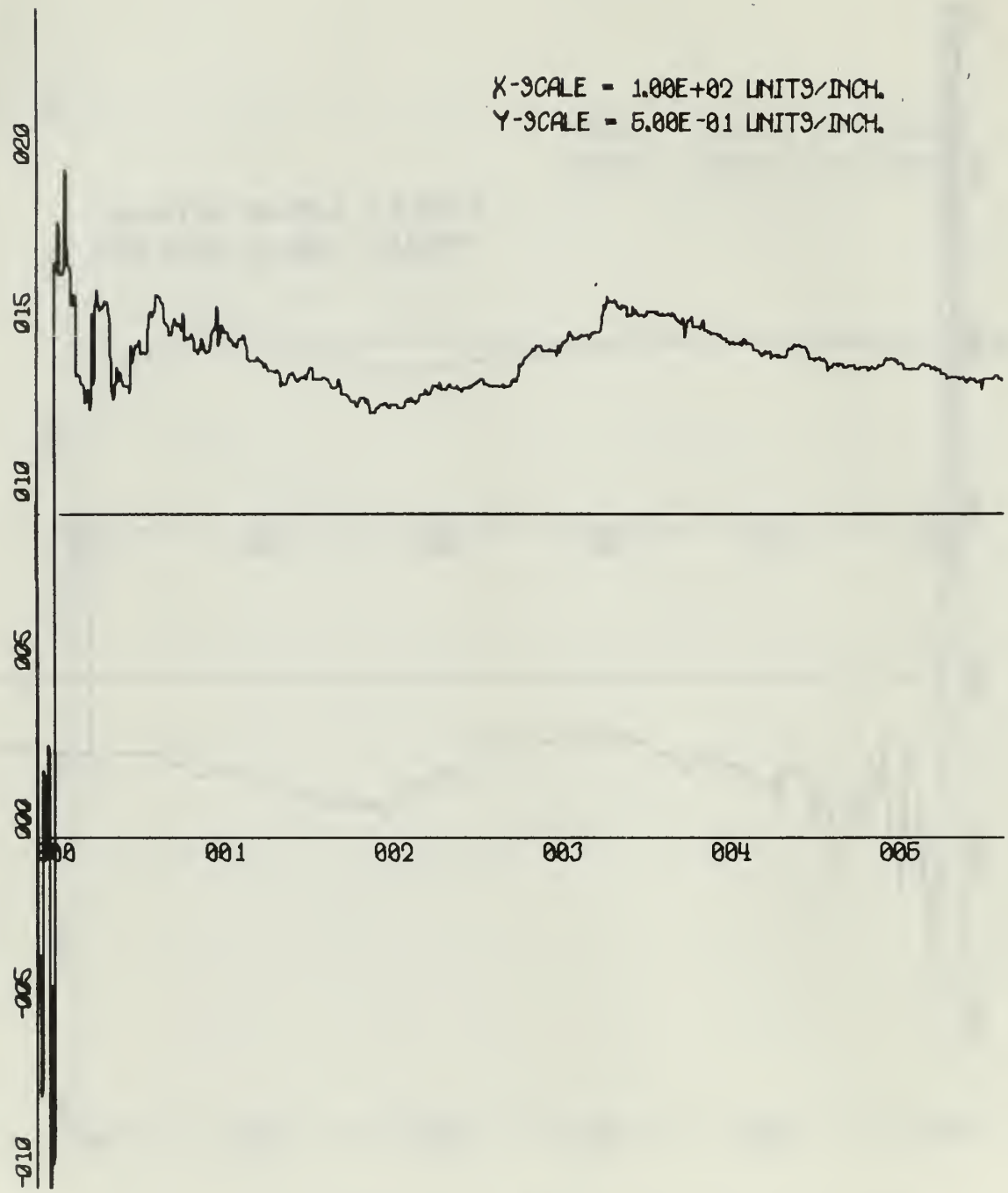
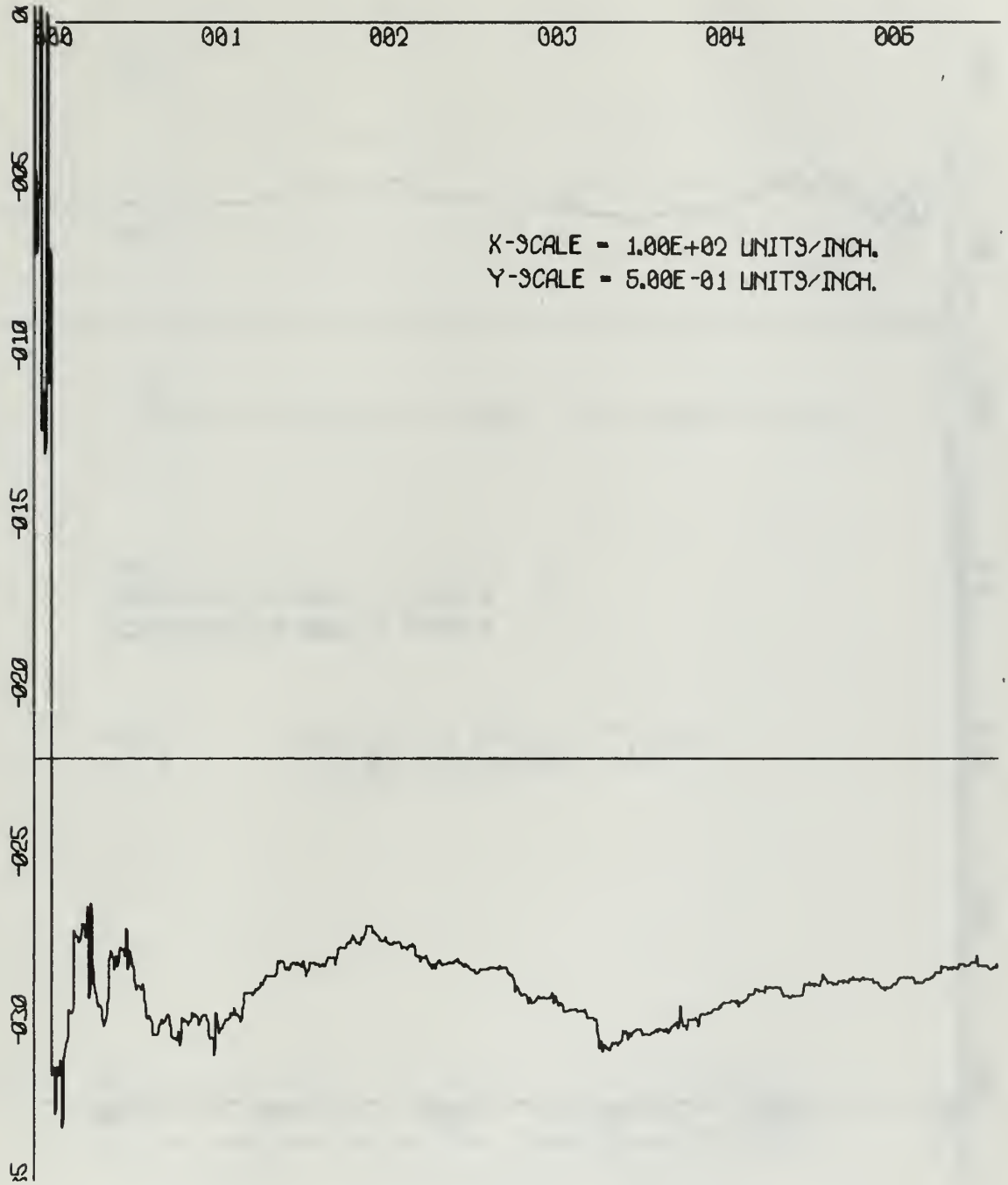


Fig. 5. Recursive Convergence of $\hat{\phi}^*$ vs Sample Number, No Noise using the 10 sec Dominant Period Plant, Zero Initial Conditions. Sampled and Excited at 1 sec Intervals, $\gamma = 0.7$. The Straight Line Represents the True Value of the Particular ϕ^* Element. The Full Γ^* vector was used to Demonstrate the Residues from Numerator Dynamics. (a) $\hat{\phi}^*(4,1)$ vs Sample Number. (b) $\hat{\phi}^*(4,2)$ vs Sample Number. (c) $\hat{\phi}^*(4,3)$ vs Sample Number. (d) $\hat{\phi}^*(4,4)$ vs Sample Number.

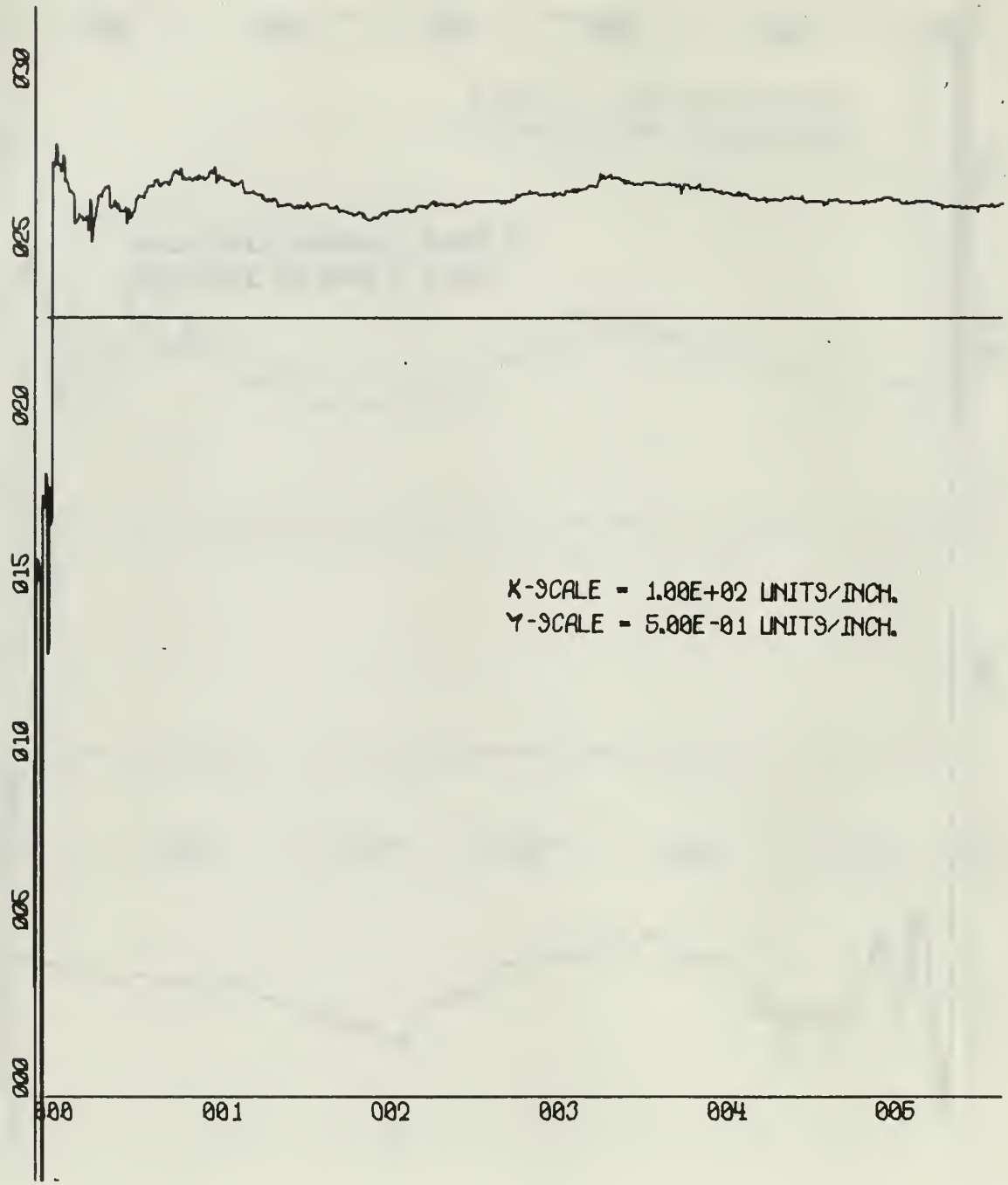
(a) $\hat{\phi}^*(4,1)$ vs Sample Number, ($\phi^*(4,1) = -0.17217$)



(b) $\hat{\phi}$ *(4,2) vs Sample Number, (ϕ *(4,2) = 0.96328)



(c) $\hat{\phi}$*(4,3) vs Sample Number, (ϕ*(4,3) = -2.1772)



(d) $\hat{\phi}$ *(4,4) vs Sample Number, (ϕ *(4,4) = 2.3215)

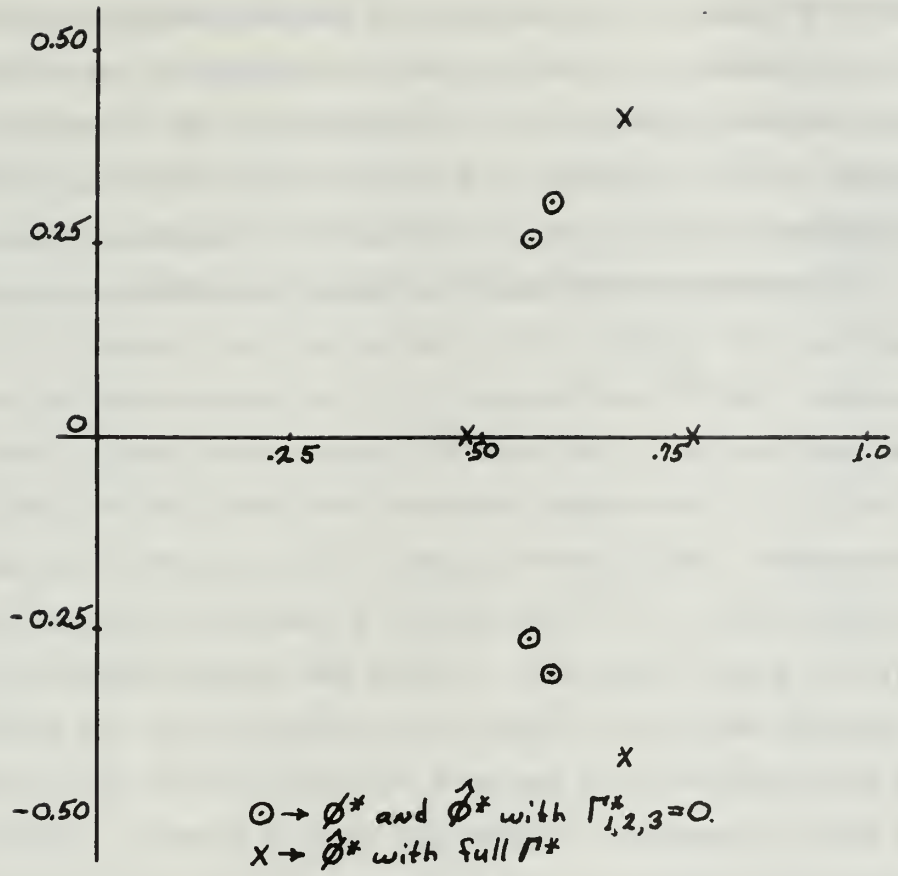


Fig. 6. Eigenvalues for ϕ^*, $\hat{\phi}^*$ with $\Gamma^*_{1,2,3} = 0$, and $\hat{\phi}^*$ using the Complete Γ^* Resulting in Residues for the 10 sec Dominant Period Plant Sampled and Excited at 1 sec Intervals. No Measurement Noise.

SAMPLE RATE

The choice of sample rate for discrete stochastic simulation does not necessarily follow the supposition that the more rapidly a system is sampled the "better". In some cases the existence of an optimal sample rate has been shown [6] . To investigate the accuracy of identification by batch processing, a sample set of 500 was generated for sample intervals from 0.02 to 4.0 seconds in increments of 0.02 seconds. The model was excited by a different excitation set for each sample rate but with the same statistical properties, normal (0,1) "white" noise. The N^{th} row elements of $\hat{\phi}^*$ for each sample set were subtracted from the true ϕ^* values and plotted versus sample interval, fig. 7a,b,c,d. For sample intervals less than 0.06 seconds R(0) went singular. Each element appears to have its own best sample rate ranging from 1 to 1.5 seconds for a dominant 10 second plant. Hence a rule of thumb: Sample at 10 times the dominant frequency!

To eliminate the effect of varying the excitation set, the above procedure was repeated using the same excitation set for each sample period, 0.05 to 10 seconds in increments of 0.05 seconds. The resulting error curves were the same as illustrated by the $\hat{\phi}^*(4,2)$ curve, fig. 8a. The apparent stabilization of the error as the sample period approaches the natural period of the system is misleading. The absolute values of ϕ^* become small due to the large damping coefficient but percent errors actually increase.

It is interesting to note that as the dominant period is changed the shape of the error curve is the same but shifted in the direction of change of the period. The ratios of sample period to natural period remain constant at the zero error intercept points. The error curves for the $\hat{\phi}^*(4,2)$ element for natural periods of 8 and 6 seconds are shown in fig. 8b,c. All of these curves were generated using the full Γ^* vector and hence contain numerator dynamics and their inherent residues. Thus it is pertinent that there is some optimal sample rate that minimizes residue errors.

Errors are not due solely to numerator dynamics. The original contention that the choice of sample period is significant may be emphasized by repeating the above runs for the ten second plant with suppressed Γ^*. The resulting identification errors are plotted versus sample period for each element, fig. 9a,b,c,d. Errors may be significantly different if the excitation, W_k, drives the system at a different rate than the sampling (observation) frequency.

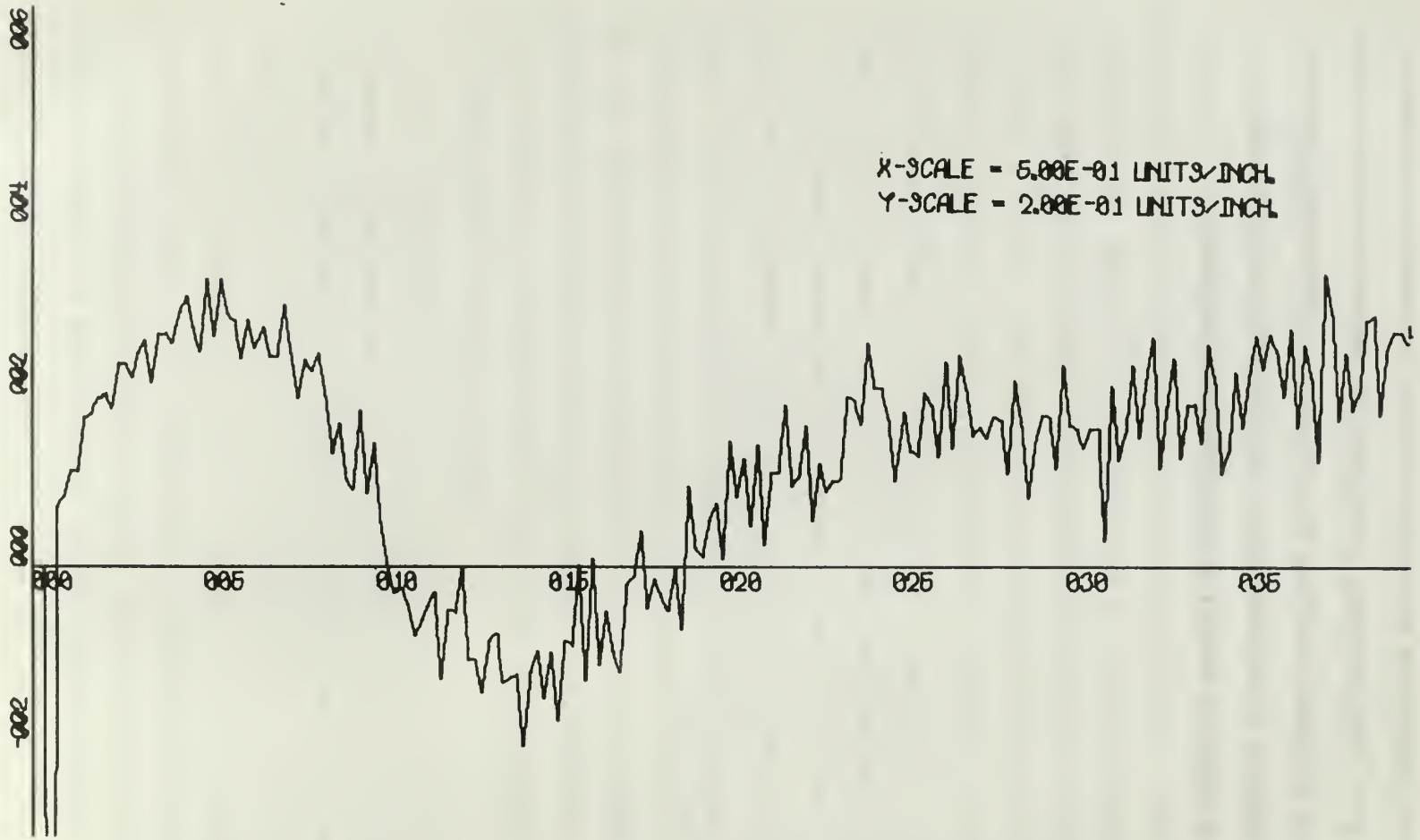
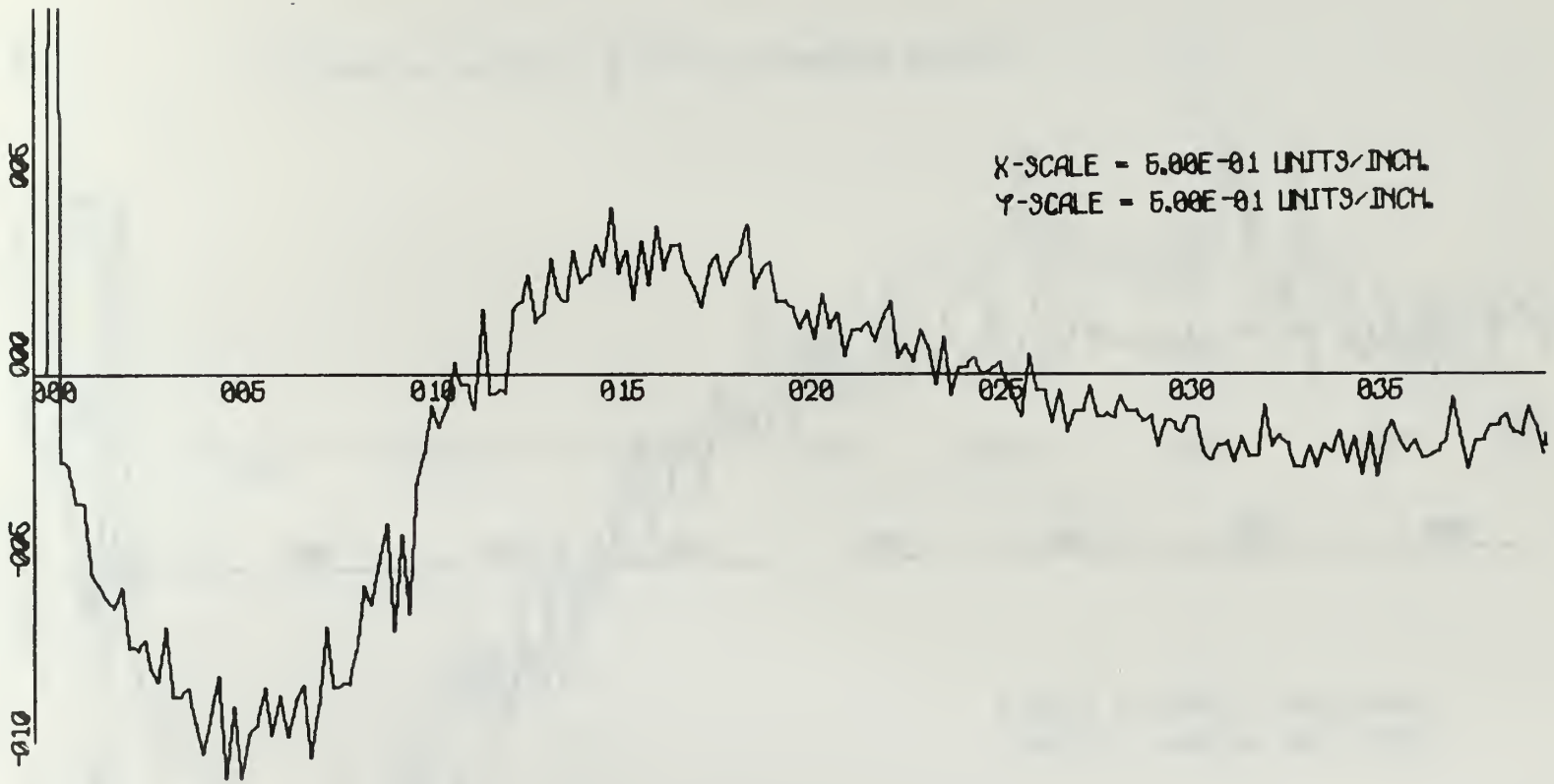
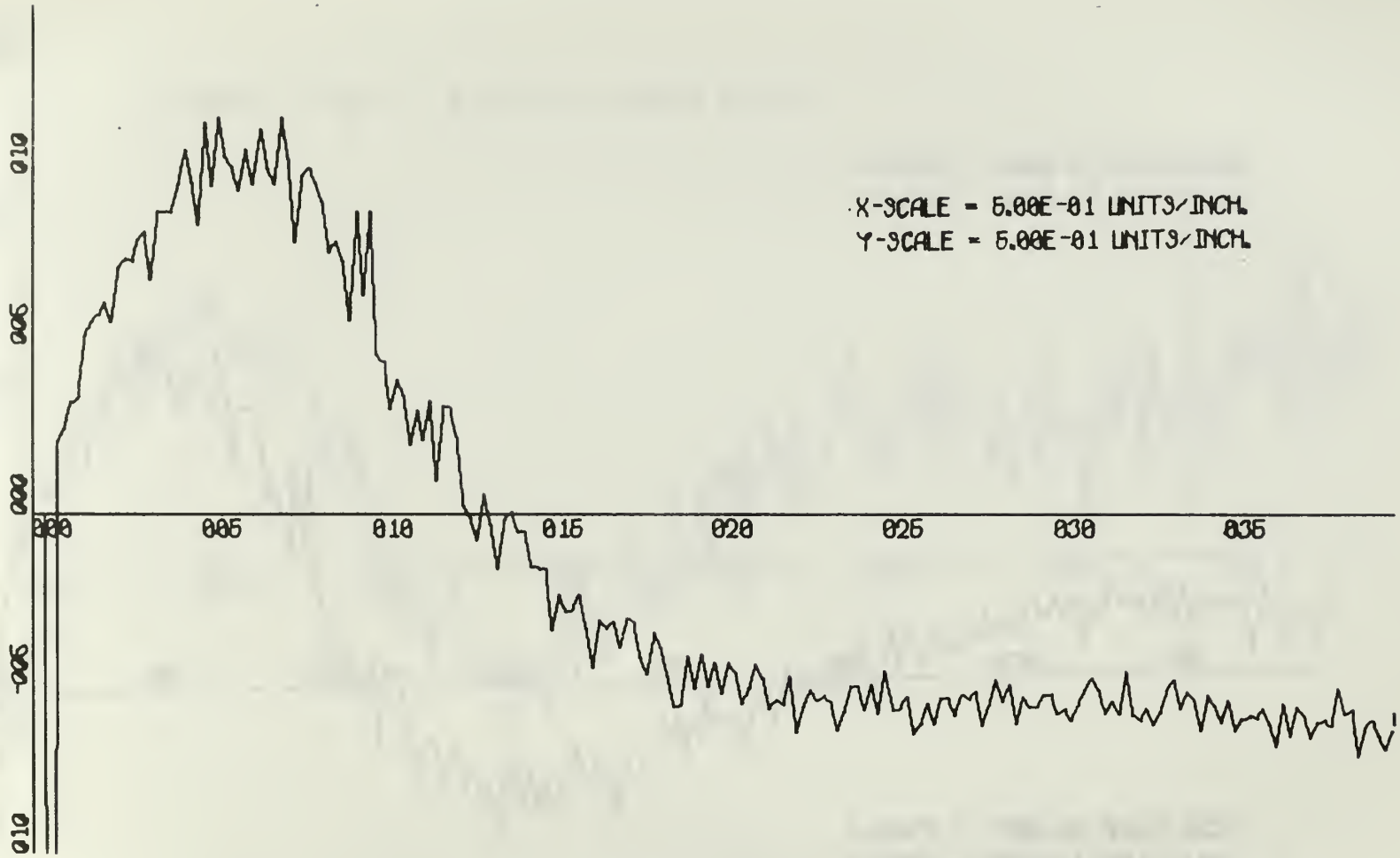


Fig. 7. Error = $\phi^* - \hat{\phi}^*$ vs Sample Period (0.02 to 4.0 sec) using Batch Processing of 500 Samples from the 10 sec Dominant Period Plant. Discrete Gaussian Excitation, $\sigma = 20$, was applied at the Sample Rate. A Different Excitation was used for each Sample Period. Γ^* was not Suppressed. (a) Error = $\phi^*(4,1) - \hat{\phi}^*(4,1)$ vs Sample Period. (b) Error = $\phi^*(4,2) - \hat{\phi}^*(4,2)$ vs Sample Period. (c) Error = $\phi^*(4,3) - \hat{\phi}^*(4,3)$ vs Sample Period. (d) Error = $\phi^*(4,4) - \hat{\phi}^*(4,4)$ vs Sample Period.

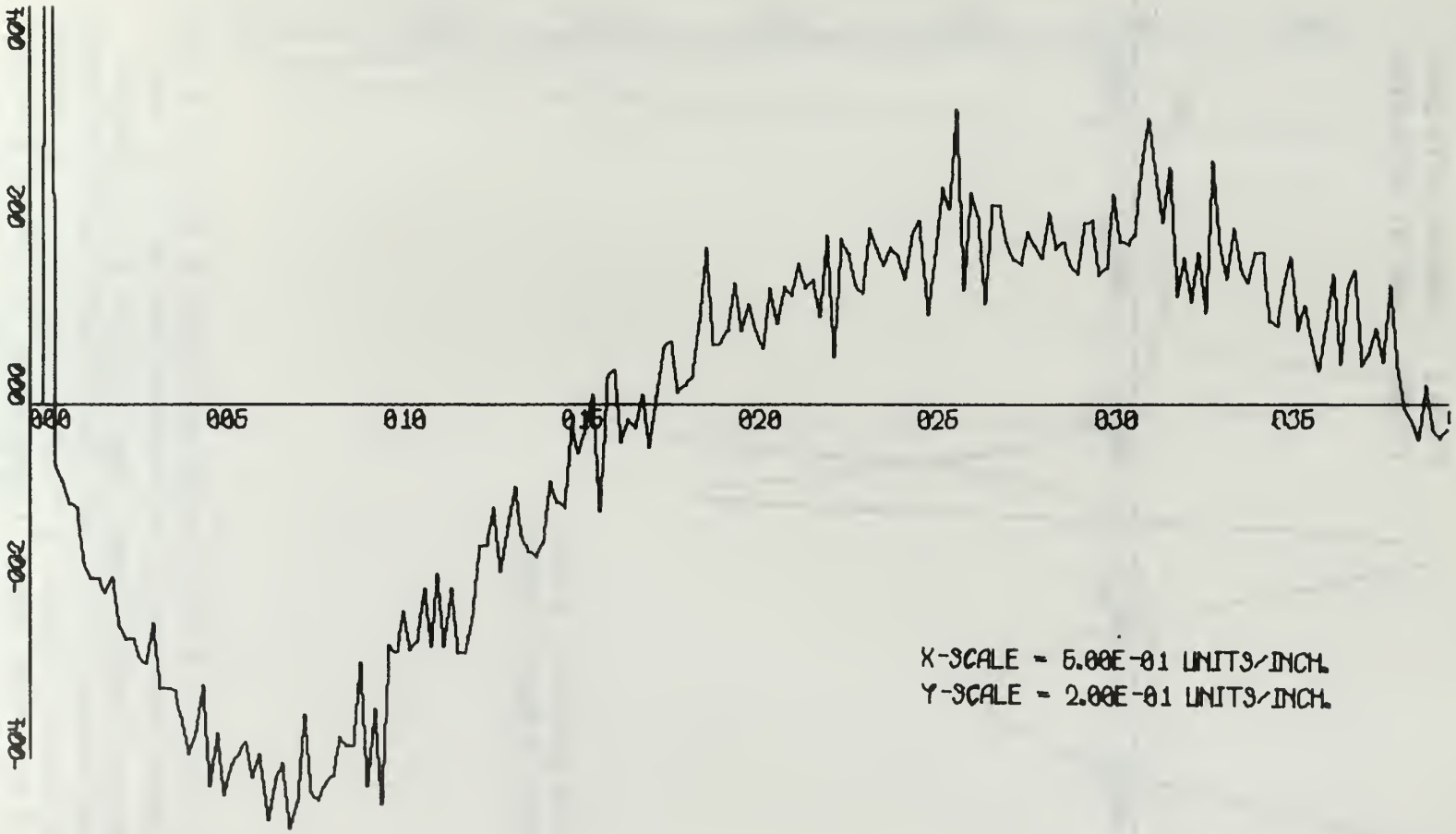
(a) Error = $\phi^*(4,1) - \hat{\phi}^*(4,1)$ vs Sample Period.



(b) Error = ϕ *(4,2) - $\hat{\phi}$ *(4,2) vs Sample Period.



(c) Error = ϕ *(4,3) - $\hat{\phi}$ *(4,3) vs Sample Period.



(d) Error = $\phi^*(4,4) - \hat{\phi}^*(4,4)$ vs Sample Period.

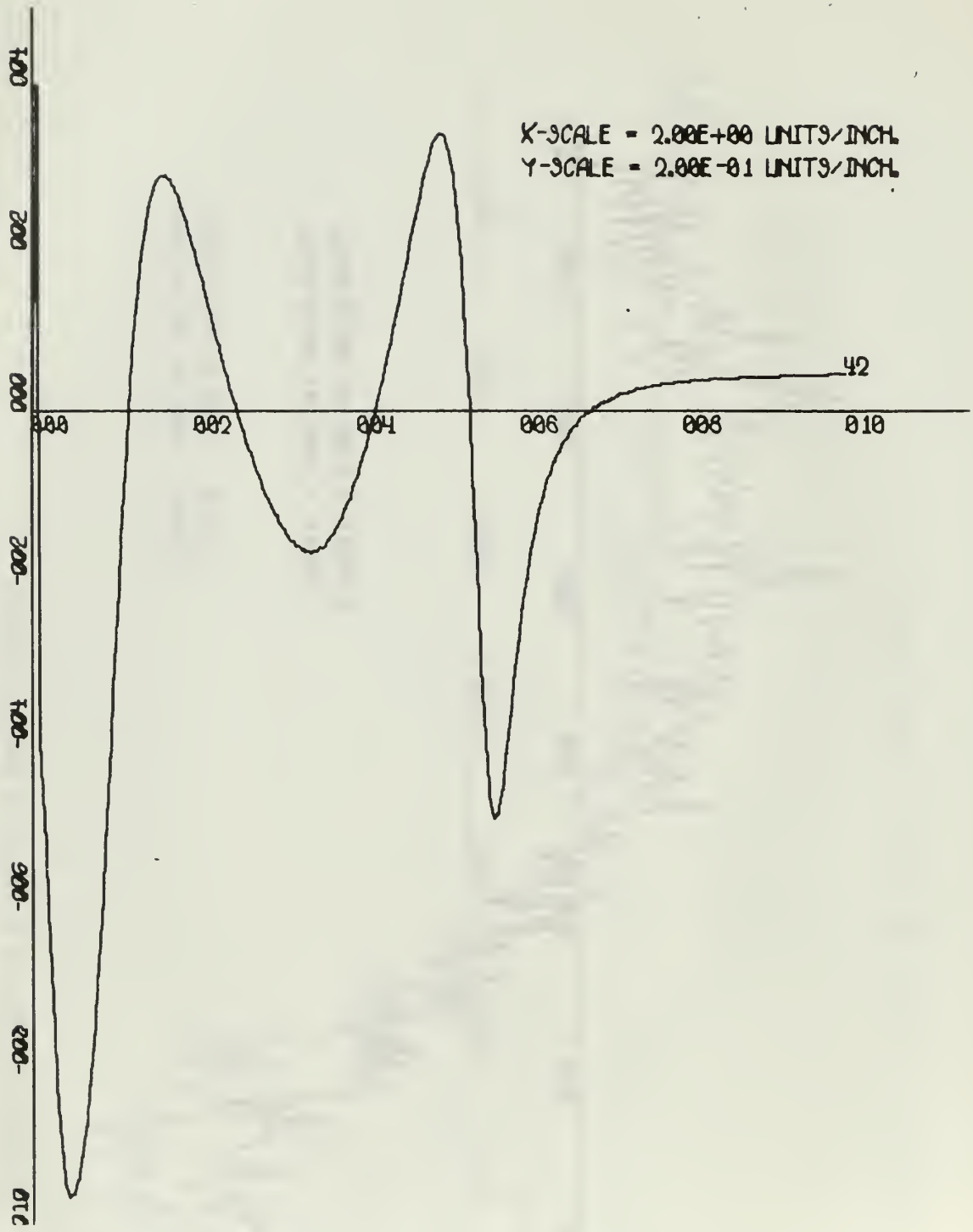
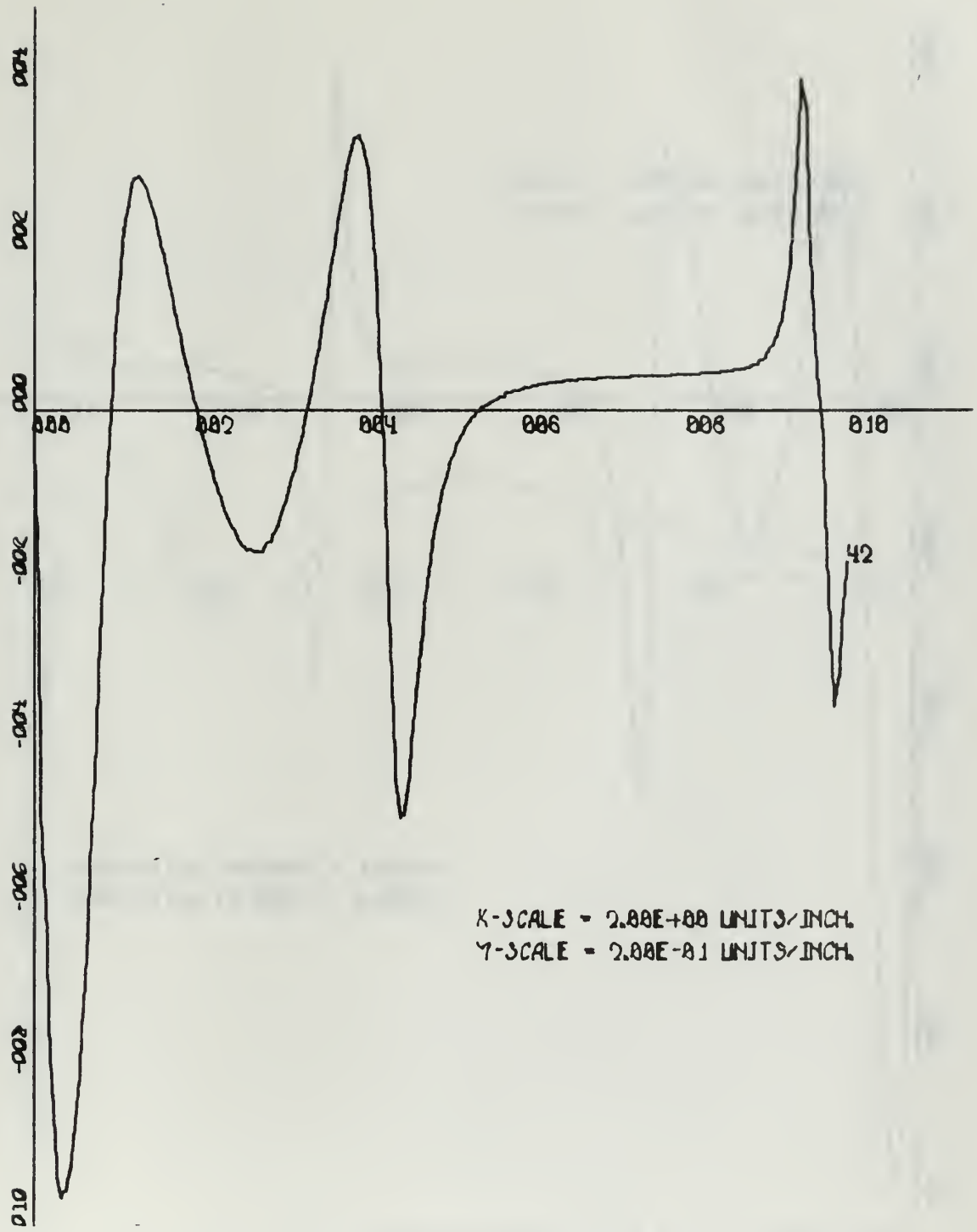
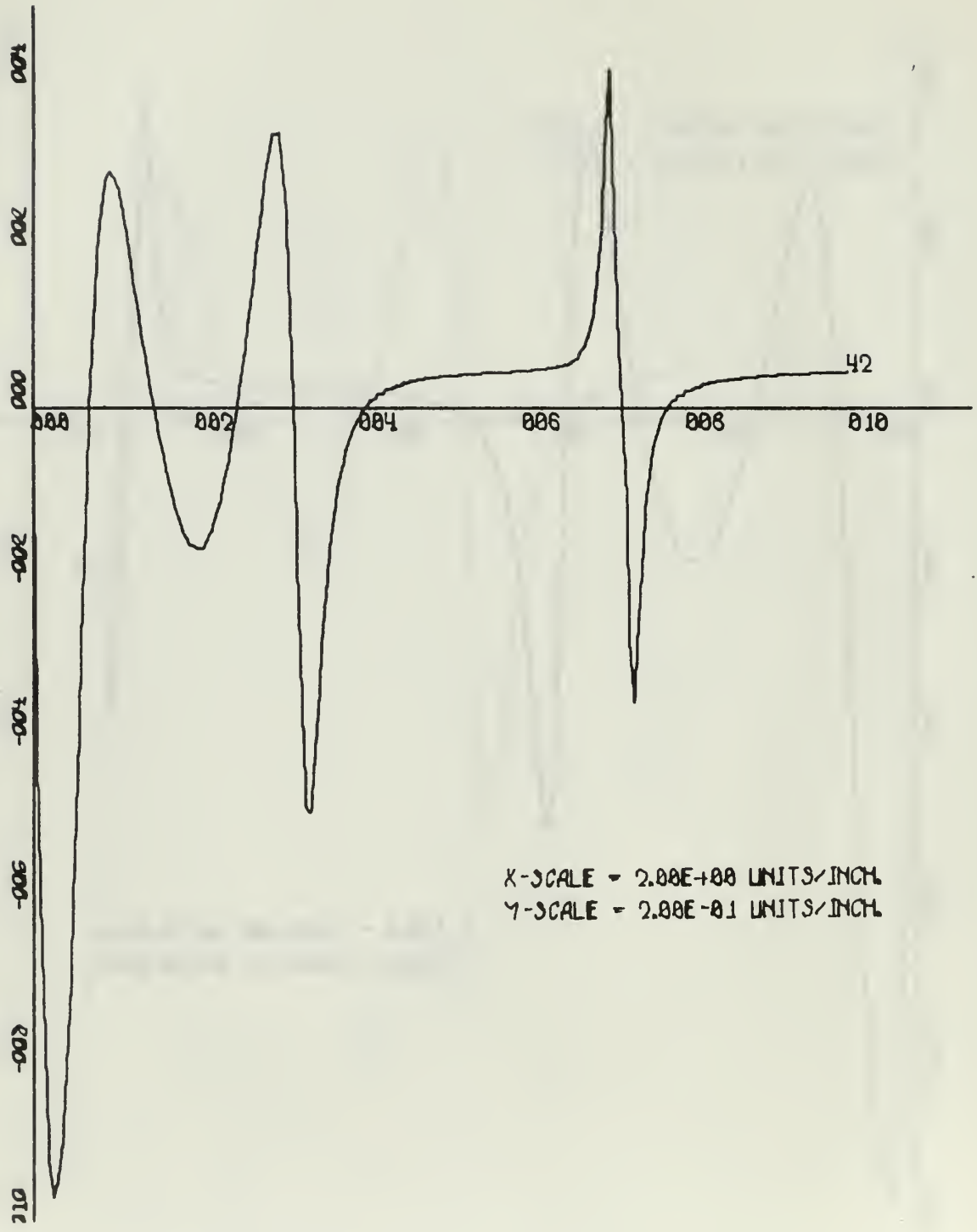


Fig. 8. Error = $\phi^*(4,2) - \hat{\phi}^*(4,2)$ vs Sample Period (0.05 to 10.0 sec) using Batch Processing of 500 Samples from the 10, 8, and 6 sec Dominant Period Plants. One Sequence of 500 Discrete Gaussian Values, $\sigma = 20$, was used as Excitation for all Sample Periods and all 3 Plants. Γ^* was not Suppressed. (a) Error of $\hat{\phi}^*(4,2)$ vs Sample Period, 10 sec Plant. (b) Error of $\hat{\phi}^*(4,2)$ vs Sample Period, 8 sec Plant. (c) Error of $\hat{\phi}^*(4,2)$ vs Sample Period, 6 sec Plant.

(a) Error of $\hat{\phi}^*(4,2)$ vs Sample Period (sec), 10 sec Plant.



(b) Error of $\hat{\phi}$ *(4,2) vs Sample Period (sec), 8 sec Plant.



(c) Error of $\hat{\phi}$ *(4,2) vs Sample Period (sec), 6 sec Plant.

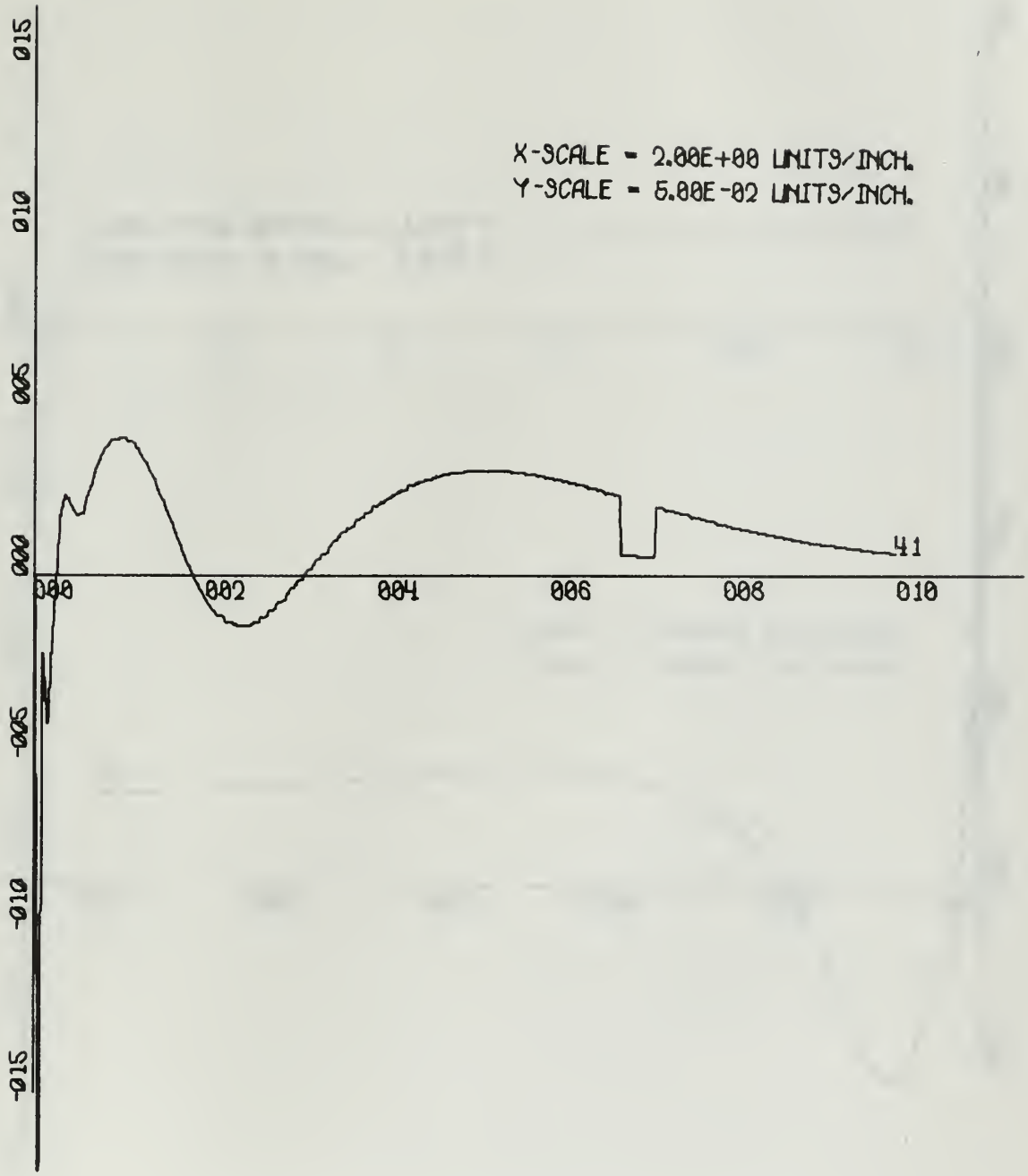
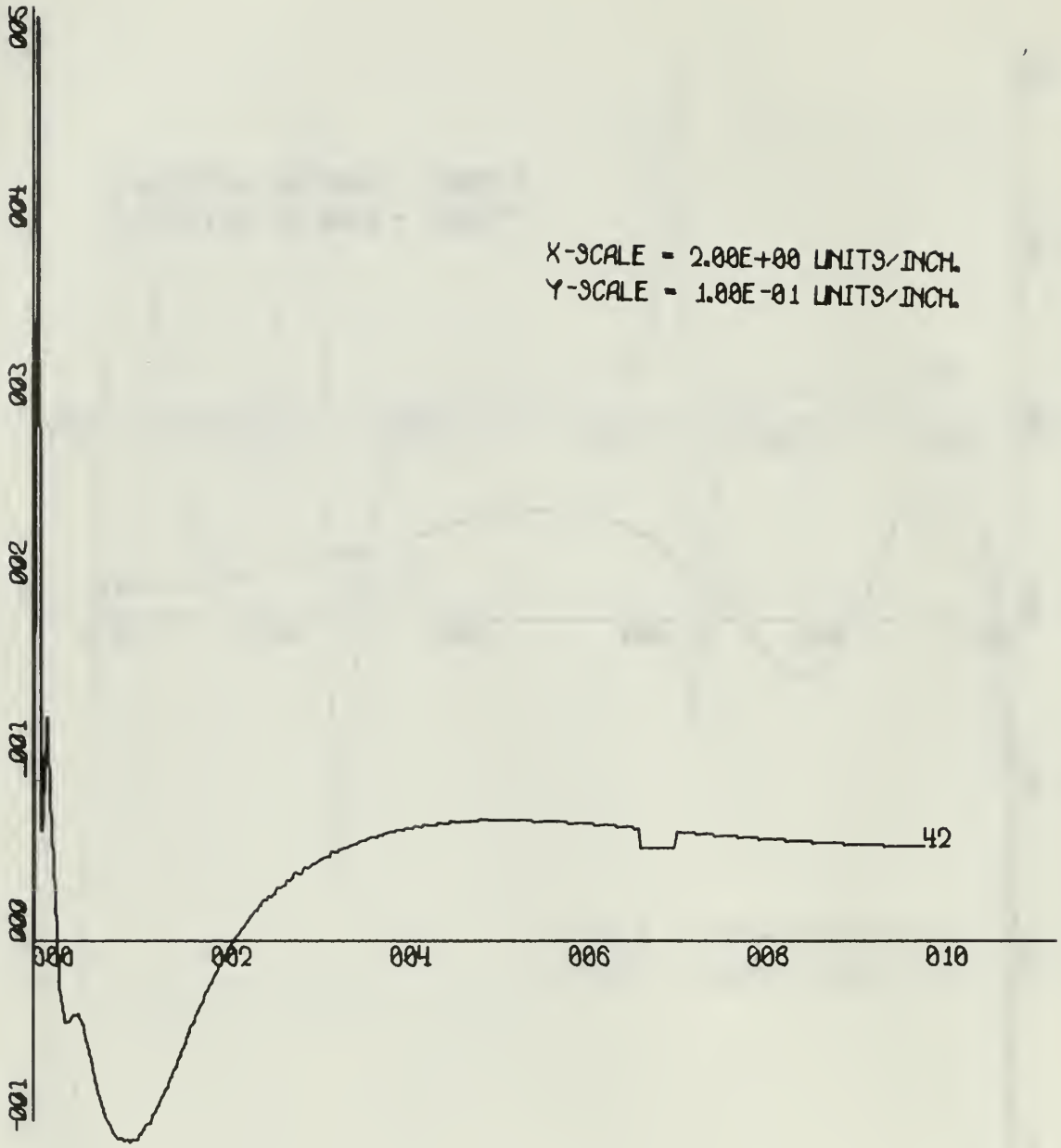
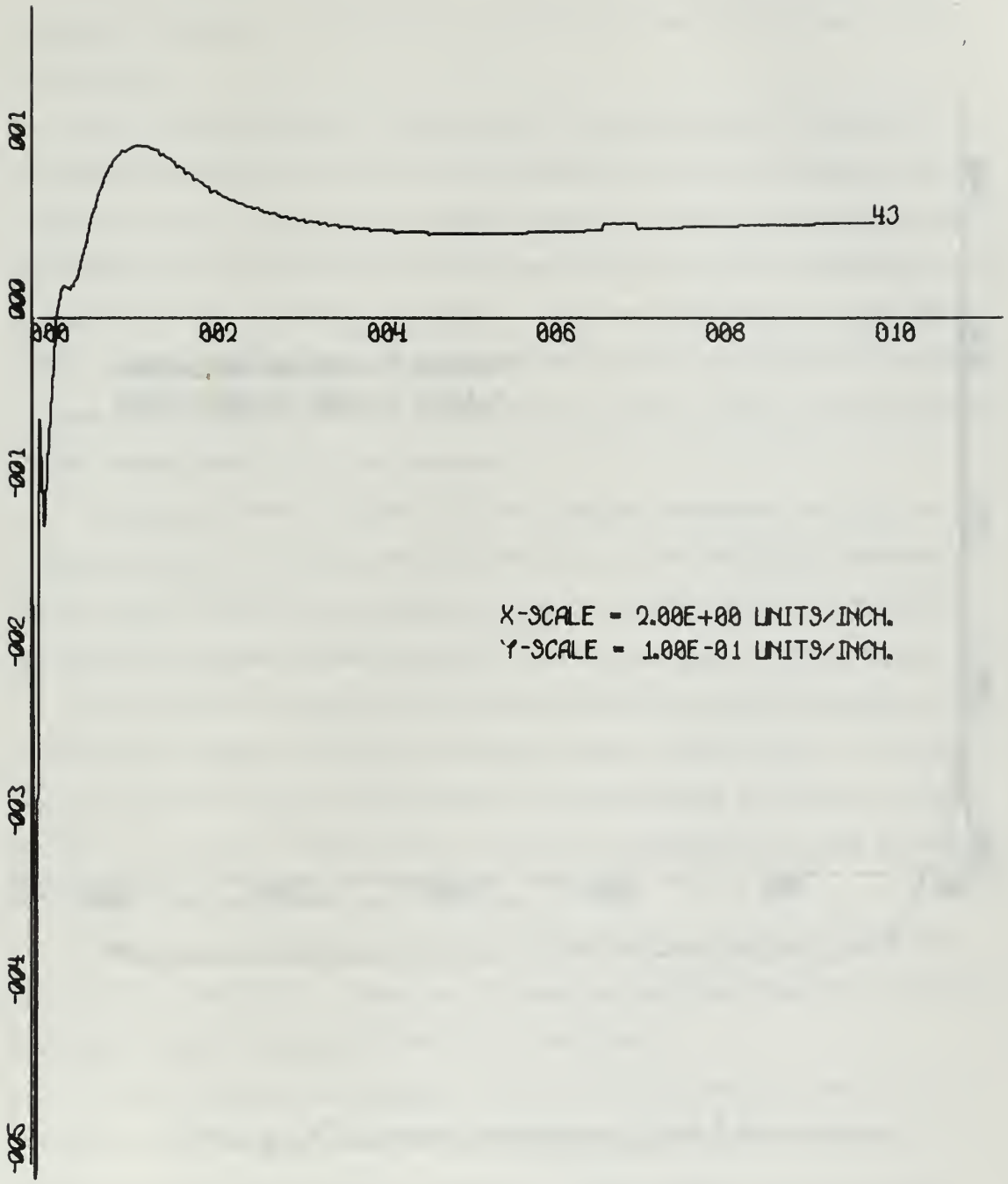


Fig. 9. Error = ϕ^* - $\hat{\phi}^*$ vs Sample Period (0.05 to 10.0 sec) using Batch Processing of 500 Samples from the 10 sec Dominant Period Plant with Γ^* Suppressed (Γ^* 1,2,3 = 0). Discrete Gaussian Excitation, σ = 20, was applied at the Sample Rate. The Same Excitation Set was used for each Different Sample Period. (a) Error of $\hat{\phi}^*(4,1)$ vs Sample Period. (b) Error of $\hat{\phi}^*(4,2)$ vs Sample Period. (c) Error of $\hat{\phi}^*(4,3)$ vs Sample Period. (d) Error of $\hat{\phi}^*(4,4)$ vs Sample Period.

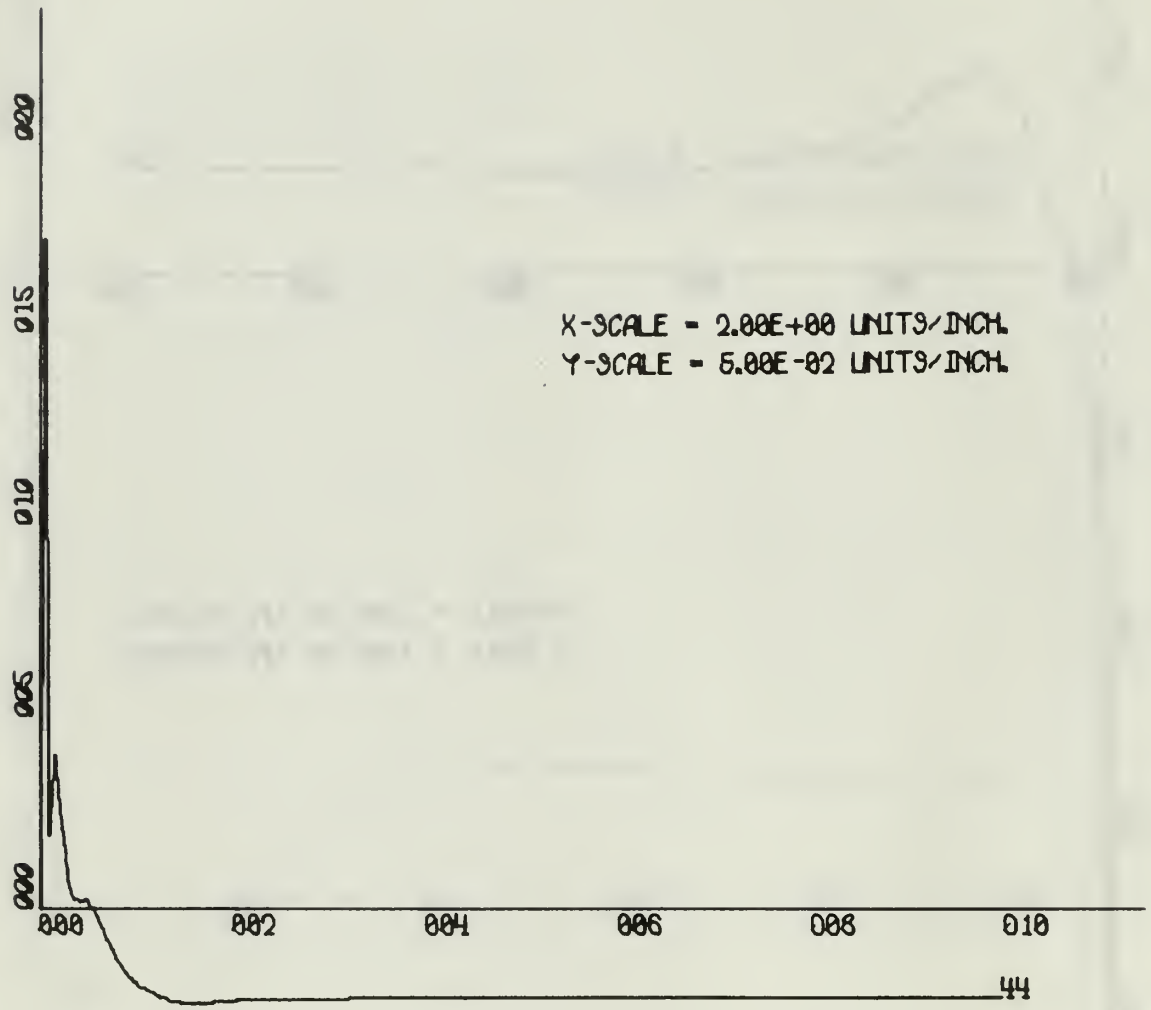
(a) Error of $\hat{\phi}^*(4,1)$ vs Sample Period, $\Gamma^*_{1,2,3} = 0$



(b) Error of $\hat{\phi}^*(4,2)$ vs Sample Period, $\Gamma^*_{1,2,3} = 0$



(c) Error of $\hat{\phi}^*(4,3)$ vs Sample Period, $\Gamma^*_{1,2,3} = 0$.



(d) Error of $\hat{\phi}^*(4,4)$ vs Sample Period, $\Gamma^*_{1,2,3} = 0$.

APPLICATION TO SHIP MOTION ESTIMATION

As a result of the foregoing work and associated simulation, the following proposal is offered as a practical procedure for ship motion estimation.

Initial identification of the order of the pitch and roll plants and the corresponding ϕ *'s is to be evaluated for the free dynamic system with the ship in calm water. Initial angle may be applied with weights or other means. The data thus obtained may be used as a reference and to initialize the stochastic routines. The yaw plant may not be identified as a free dynamic system since there is no suitable way to apply a controlled initial condition which will project on all the eigenvectors in the absence of a righting moment.

Stochastic identification is to be applied whenever the ship is underway at sea. Either batch processing or the recursion equations may be used. Batch processing should be used at least periodically to verify the system order from $R(0)^{-1}$ and to identify Q if required.

The recursion equations are more suitable for time sharing and do not require storage to accumulate large sample sets of data. Since periodic identification requirements are anticipated to follow system dynamics, recursive identification may be initiated using the previous $\hat{\phi}$* as the best estimate and flagging in P(0).

The actual estimation of ship attitude follows the identification of the ϕ *'s (and Q's). Equation (9) may be applied directly to predict. Discrete linear (Kalman) filtering is available.

The development throughout this paper has assumed that the only reliable or convenient measurements available for identification were discrete pitch, roll, and yaw angles. If the number of observables is increased, the accuracy of the identifications and the estimations may be expected to improve [7].

CONCLUSIONS

1. Identification of ship motion dynamics is feasible by stochastic methods.
2. The accuracy of the identification is dependent upon the numerator dynamics of the true system z-transfer function. The contention of Lee [4] that decorrelation would improve accuracy was not true for this work or that of Blackner [7] .
3. The accuracy is dependent upon the sample period. An optimal sample rate exists.
4. Identification of an unexcited plant (the free dynamic system) having a single initial condition is independent of Γ^* and hence is not affected by numerator dynamics. This should be the source of the most reliable identification as a reference.
5. Further work should be done using actual ship motion data to complete the identification evaluation and to investigate the performance of discrete linear (Kalman) filtering as the final stage of ship motion estimation.

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* Translations available from Fleet Numerical Weather Facility, Monterey, California.

APPENDIX

CDC FORTRAN 63 Programs and Subroutines for Digital Simulation and Identification.

PHITEST: Main program generating the simulation model and controlling the identification.

PHIDENT: Recursive identification routine for the scalar observable case.

PHICALL: Batch processing for general observable case. Identifies ϕ *, N, and Q. (GAUSS3 is identical to RECIP)

PHIDEL: Computes standard form ϕ and Γ matrices for sample data systems.

PHIDEL1: Computes ϕ * and Γ * matrices for sample data systems.

PSD: Computes Power Spectral Density from auto-correlation functions.

RECIP: Matrix inversion routine.

ADD, SUB, TRANS, PROD, PRINTER, and MATREAD: Convenient subroutines for the indicated matrix operations.

PTPLOT: Graphs and prints out 2 dimensional data on line printer.

```
      PROGRAM PHITEST
      DIMENSION R(500),S(500),U(500),V(5  ),IT(12)
      DIMENSION Q(4,4),QUE(4,4),DELT(1,4), TE(1,4),EWK(1,1),EXCITE(500)
     1FHI(4,4),ADWK(500)
      DIMENSION PHI(4,4),DEL(4,1),PHI1(4,4),DEL1(4,1),
     1A(4,4),B(4,1),H(4,4),TT(501),PH(10,10),
     2PK(10,10),X1(500),X2(500),X3(500),X4(500)
      DIMENSION Y(1,500)
C
C     WN=NATURAL FREQUENCY * 2PI
C     F  = FREQUENCY CYCLES/SEC.
C     ZETA = DAMPING COEFFICIENT
C     PERIOD = NATURAL SYSTEM PERIOD (SEC.)
C
C     GIVEN THE SET OF STATE DIFFERENTIAL EQUATIONS
C       XDOT = AX + BU  FOR AN NTH ORDER SYSTEM.
C
C     A  = N * N COEFFICIENT MATRIX (ASSUMED CONSTANT)
C
C     B = N * 1 MATRIX OF EXCITATION COEFFICIENTS
C     PHI = N * N STATE TRANSITION MATRIX
C
C     PHI1 = N * N STATE TRANS. MATRIX IN CANONIC FORM
C            ***             ***
C            * 0   1   0   0 *
C     PHI1 = * 0   0   1   0 * = PHI*
C            * 0   0   0   1 *
C            * A   B   C   D *
C            ***             ***
C
C     DEL AND DEL1 ARE THE RESPECTIVE EXCITATION COEFFICIENTS
C
C     THE ABOVE RELATIONSHIPS FORM THE STATE DIFFERENCE EQNS.
C       X(K+1) = PHI*X(K) + DEL*W(K)  OR
C       Y(K+1) = PHI1*Y(K) + DEL1*W(K)
```

```
C          WHERE W(K) IS GAUSSIAN
C        Y1,1(K) = X1,1(K)
C
      DO 7 I=1,12
    7 IT(I)=8H
      IT(1)=8H100 RUNS
      IT(7)=8HMCDONOUG
      IT(8)=8HH, T.F.
      IT(9)=8H APR 67
      IT(10)=8HSHIP MOT
      IT(11)=8HION ESTI
      IT(12)=8HMATION
      N=4
      K50 =499
      EP= .000000001
      DO 25 I=1,500
   25 TT(I)=I
      PERIOD=6.
      PERIOD=8.
      PERIOD=10.
      T=0.05
      F=1./PERIOD
      ZETA=0.7
      JJ=200
      DO 777 LL=1,JJ
      NUNIF=7293184363
      DO 1 I=1,4
      DEL1(I,1)=0.
      DEL(I,1)=0.
      B(I,1)=0.
      DO 1 J=1,4
      PHI1(I,J)=0.
      PHI(I,J)=0.
      H(I,J)=0.
    1 A(I,J)=0.
```

```
      B(4,1)=1.
      DO 2 I=1,3
    2 A(I,I+1)=1.
      H(1,1)=1.
      WN=2.*(4.*ATANF(1.))*F
      A(4,1)=-1.*WN**4
      A(4,2)=-4.*ZETA*WN**3
      A(4,3)=-4.*(ZETA*WN)**2-2.*WN**2
      A(4,4)=-4.*ZETA*WN
C
C     PHIDEL OUTPUTS PHI* AND DEL* INTO MATRICES LABELED PHI AND DEL
      CALL PHIDEL1(A,B,H,N,1,4,1,T,PHI,DEL)
C
C     SUPPRESS UNWANTED ELEMENTS OF DEL TO ELIMINATE NUMERATOR DYNAMICS
      DO 335 I=1,3
  335 DEL(I,1)=0.
C
C     INITIAL CONDITIONS
      X1(1)=0. $ X2(1)=0. $ X3(1)=0. $ X4(1)=0.
      Y(1,1)=0.
      DO 6 I=1,K500
C
C     EXCITATION FROM RANDOM NUMBER ROUTINE
      CALL RNDEV63(NUNIF,DEV)
      WK=DEV*20.
C
C     FOURTH ORDER PLANT TO GENERATE SAMPLE SETS
      X1(I+1)=PHI(1,1)*X1(I)+PHI(1,2)*X2(I)+PHI(1,3)*X3(I)+
     1PHI(1,4)*X4(I)+DEL(1,1)*WK
      X2(I+1)=PHI(2,1)*X1(I)+PHI(2,2)*X2(I)+PHI(2,3)*X3(I)+
     1PHI(2,4)*X4(I)+DEL(2,1)*WK
      X3(I+1)=PHI(3,1)*X1(I)+PHI(3,2)*X2(I)+PHI(3,3)*X3(I)+
     1PHI(3,4)*X4(I)+DEL(3,1)*WK
      X4(I+1)=PHI(4,1)*X1(I)+PHI(4,2)*X2(I)+PHI(4,3)*X3(I)+
     1PHI(4,4)*X4(I)+DEL(4,1)*WK
```

```
      Y(1,I+1)=X1(I+1)
      IB=I+1
    6 CONTINUE
C
C       DRAW SELECTED ROLL ANGLES IF DESIRED
      IF(T-0.1)21,20,21
   20 LA=4H
      CALL DRAW(IB,TT,X1,0,0,LA,IT,0,0,0,0,0,0,6,6,0,LAST)
   21 CONTINUE
      NK=N
C
C       PHICALC BATCH PROCESSES TO IDENTIFY
C       PHI*HAT = FHI FROM PHICALC
      CALL PHICALC(Y,NK,IB ,FHI,QUE,1,50 ,N ,1)
      IF(NK-4)4,5,4
    4 PRINT 3,LL
    3 FORMAT(/,2X,23HRO WENT SINGULAR ON RUN ,I4)
      N=4
      IF(LL.EQ.1)12,11
   11 R(LL)=R(LL-1) $ S(LL)=S(LL-1) $ U(LL)=U(LL-1) $ V(LL)=V(LL-1)
      GO TO 13
    5 CONTINUE
C
C       ERRORS FOR EACH ELEMENT
      R(LL)=PHI(N,1)-FHI(N,1)  $  S(LL)=PHI(N,2)-FHI(N,2)
      U(LL)=PHI(N,3)-FHI(N,3) $ V(LL)=PHI(N,4)-FHI(N,4)
      GO TO 13
   12 CONTINUE
      R(1)=0. $ S(1)=0. $ U(1)=0. $ V(1)=0.
   13 CONTINUE
C
      ADWK(LL)=T
      T=T 0.05
      IA=LL
  777 CONTINUE
```

```
      CALL PRINTER(FHI,N,N,4,4,8HPHI* PC )
      CALL PRINTER(PHI,N,N,4,4,8HPHI* TRU)
      CALL PRINTER(QUE,N,N,4,4,8HQUE PC  )
C
C     DRAW A GRAPH OF ERRORS VS SAMPLE PERIOD
      IT(1)=8HSENSITIV
      IT(2)=8HITY OF I
      IT(3)=8HDENTIFIC
      IT(4)=8HATION TO
      IT(5)=8H THE SAM
      IT(6)=8HPLE RATE
      LA=4H41
      CALL DRAW(IA,ADWK,R,0,0,LA,IT,0,0, ,0,0,0,6, 9,0,LAST)
      LA=4H42
      CALL DRAW(IA,ADWK,S,0,0,LA,IT,0,0, ,0,0,0,6, 9,0,LAST)
      LA=4H43
      CALL DRAW(IA,ADWK,U,0,0,LA,IT,0,0, ,0,0,0,6, 9,0,LAST)
      LA=4H44
      CALL DRAW(IA,ADWK,V,0,0,LA,IT,0,0, ,0,0,0,6, 9,0,LAST)
      END
```

```
      SUBROUTINE PHIDENT(PH,Z,X,N,P,IFLAG)
      DIMENSION PH(N,1),X(N),P(N,N),T(10),TT(10)
C
C     LEE HO METHOD
C
C       PH = (N,1) VECTOR. PH TRANSPOSE CORRESPONDS
C            TO THE NTH ROW OF THE PHI* MATRIX.
C        Z = MOST RECENT SCALAR MEASUREMENT = Z(K)
C        X = (Z(K-N) Z(K-N+1) ... Z(K-1) ) = (1*N) MATRIX
C        N = SYSTEM ORDER
C     IFLAG = INITIALIZE P AND PH IF NOT EQUAL TO 1
C     THE X MATRIX MUST BE LOADED PRIOR TO CALLING THIS
C     S/R. HENCE AT LEAST N MEASUREMENTS MUST BE AVAILABLE.
C
      IF(IFLAG.EQ.1)1,7
    1 C=1.
      W=Z
      DO 3 I=1,N
      T(I)=0.
      DO 2 J=1,N
      T(I)=T(I)+P(I,J)*X(J)
    2 C=C P(I,J)*X(I)*X(J)
    3 W=W X(I)*PH(I,1)
      IF(ABSF(C)-(0.1**10))10,10,33
   33 DO 4 I=1,N
      T(I)=T(I)*W/C
    4 PH(I,1)=PH(I,1)+T(I)
C
C     PH COMPLETE.  NOW GENERATE P FOR NEXT RUN.
C
      DO 5 I=1,N
      T(I)=0. $ TT(I)=0.
      DO 5 J=1,N
      T(I)=T(I)+X(J)*P(J,I)/C
    5 TT(I)=TT(I)+P(I,J)*X(J)
```

```
      DO 6 I=1,N $ DO 6 J=1,N
    6 P(I,J)=P(I,J)-TT(I)*T(J)
      RETURN
C
C     INITIALIZE P AND PH.
C
    7 DO 9 I=1,N
      DO 8 J=1,N
    8 P(I,J)=0.
      PH(I,1)=1.
    9 P(I,I)=1.E9
      IFLAG=1
      RETURN
   10 PRINT 11,C
   11 FORMAT(/,2X,13HSINGULAR, C =,E11.4)
      IFLAG=4
      END
```

```
      SUBROUTINE PHICALC(X,N3,NN,PHI,Q,N,M,N1,IFLAG)
      DIMENSION X(N,M),PHI(N1,N1),Q(N1,N1),A(9,9),AI(9,9),R0(9,9),R1(9,
     1 9)
C  PHICALC GENERATES A STATE TRANSITION MATRIX PHI FROM THE
C  OBSERVED  SEQUENCE OF DATA X, ASSUMED TO HAVE BEEN GENERATED BY
C  A LINEAR PROCESS  AND EXCITED ONLY BY SOME UNKNOWN INITIAL
C  CONDITION AND/OR A GAUSSIAN EXCITATION
C  THE ALGORITHM  PHI=(R1)(R0 INVERSE), WHERE R0 IS THE AUTOCORRELATION
C  FUNCTION AT ZERO, R1 THE AUTOCORRELATION FUNCTION AT T, THE SAMPLING
C  INTERVAL.
C  THE COVARIANCE OF RANDOM EXCITATION Q IS CALCULATED BY THE ALGORITHM
C  Q=R0-(PHI)(R1 TRANSPOSE)
C  ARGUMENTS
C    X-THE SEQUENCE OF OBSERVATIONS, DIMENSIONED(NXM).A
C      SCALER SEQUENCE (SINGLY SUBSCRIBED VARIABLE) MAY BE USED PROVIDED
C      N IS SET EQUAL TO ONE.
C    N3- SUSPECTED ORDER OF THE PLANT. IF THE ACTUAL ORDER OF THE PROCESS
C      IS LESS THAN N3, N3 WILL BE REDUCED TO INDICATE THE PLANT ORDER
C    PHI-STATE TRANSITION MATRIX. DIMENSION(N1,N1)
C    Q-COVARIANCE OF EXCITATION. DIMENSION (N1,N))
C    IFLAG-SETTING IFLAG=1  GIVES PHI IN CANONIC FORM. IF N LESS THAN N3,
C      PHI WILL BE IN CANONIC FORM.
C    NN-NUMBER OF OBSERVATIONS. MUST BE GREATER THAN N3 FOR VECTOR
C      OBSERVATIONS, GREATER THAN 2*N3+1 FOR SCALAR OBSERVATIONS
C      OR CANONIC FORM
      EP=1.E-4
      DO 1 I=1,N1  $ DO 1 J=1,N1
      R0(I,J)=0. $ Q(I,J)=0. $ PHI(I,J)= .
    1 R1(I,J)=0.
      IF(IFLAG.EQ.1)14,20
   20 IF(N-N3)14,21,21
   21 NN=NN-1  $ XN=NN-1
      DO 2 K=1,NN  $DO 2 I=1,N3  $ DO 2 J=1,N3
      R0(I,J)=R0(I,J)+X(I,K)*X(J,K)
    2 R1(I,J)=R1(I,J)+X(I,K+1)*X(J,K)
```

```
 3 DO 4 I=1,N3 $ DO 4 J=1,N3
 4 A(I,J)=R0(I,J)
   CALL GAUSS3(N3,EP,A,AI,KER,9)
   IF(KER-1)6,6,5
 5 N3=N3-1
   IF(N3)22,22,3
 6 IF(IFLAG.EQ.1) 16,7
 7 DO 8 I=1,N3$ DO 8 J=1,N3
 8 R0(I,J)=R0(I,J)+X(I,NN+1)*X(J,NN+1)-X(I,1)*X(J,1)
 9 DO 10 I=1,N3  $ DO 10 J=1,N3 $ DO 10 L=1,N3
10 PHI(I,J)=PHI(I,J) + R1(I,L)*AI(L,J)
   DO 12 I=1, N3 $ DO 12 J=1,N3
   A=0. $ DO 11 L=1,N3
11 A=A PHI(I,L)*R1(J,L)
12 Q(I,J)=(R0(I,J)-A)/XN
   RETURN
14 NN=NN-N3 $ XN=NN-1 $ IFLAG=1
   DO 15 K=1,NN  $ DO 15 I=1,N3 $ DO 15 J=1,N3
   R0(I,J)=R0(I,J)+X(1,K+I-1)*X(1,K+J 1)
15 R1(I,J)=R1(I,J)+X(1,K+I)*X(1,J+K-1)
   GO TO 3
16 DO 17 I=1,N3 $ DO 17 J=1,N3
17 R0(I,J)=R0(I,J)+X(1,NN+I)*X(1,NN+J)-X(1,I)*X(1,J)
   GO TO 9
22 END
```

```
      SUBROUTINE PHIDEL(A,B,N,M,ND,MD,T,PHI,DEL)
C
      DIMENSION A(ND,ND),B(ND,MD),PHI(ND,ND),DEL(ND,MD),A1(10,10),
     1A2(10,10),A3(10,10),A4(10,10),A5(1 ,10)
C
C        PHIDEL  COMPUTES THE STATE TRANSITION MATRIX
C     AND THE CORRESPONDING DEL FOR A LINEAR SAMPLED-DATA
C     SYSTEM WITH SCALAR FORCING FUNCTION. GIVEN THE
C     LINEAR TIME INVARIANT DIFFERENTIAL EQUATION,
C      XDOT = AX + BU, THE PHI AND DEL FOR THE STATE
C     DIFFERENCE EQUATION, X(K+1) = PHI*X(K) + DEL*U(K),
C     ARE FOUND BY SERIES EXPANSION OF E**AT.
C
C     SYMBOLS USED.
C        XDOT       N*1 VECTOR
C        X          N*1 STATE VECTOR
C        A          N*N COEFFICIENT MATRIX
C        B          N*M COEFFICIENT MATRIX
C        U          M*1 INPUT VECTOR
C        K              SAMPLE NUMBER, K=1,2,...
C        T              FIXED TIME BETWEEN SAMPLES
C        A1,2,3,4,5     TEMPORARY STORAGE
C
      SIZE=0.1E-20
      DO 1 I=1,N
      DO 1 J=1,N
      PHI(I,J)=0.
      PHI(I,I)=1.
      A4(I,J)=0.
      A4(I,I)=T
      A5(I,J)=A(I,J)*T
    1 A1(I,J)=A(I,J)*T
      DEN=1.0
      Z=0.
    2 Z=Z 1.
```

```
      DEN=DEN*Z
      TERM=0.
      DO 4 I=1,N
      DO 4 J=1,N
      A3(I,J)=A1(I,J)/DEN
      IF(TERM-ABSF(A3(I,J)))3,4,4
    3 TERM=ABSF(A3(I,J))
    4 CONTINUE
      DO 5 I=1,N
      DO 5 J=1,N
      PHI(I,J)=PHI(I,J)+A3(I,J)
    5 A3(I,J)=A3(I,J)*DEN
      DO 6 I=1,N
      DO 6 J=1,N
    6 A1(I,J)=0.
      DO 7 I=1,N
      DO 7 J=1,N
      DO 7 K=1,N
    7 A1(I,J)=A1(I,J)+A3(I,K)*A5(K,J)
C
C        COMPUTATION OF PHI IS COMPLETE WHEN THE LAST
C     TERM OF THE SERIES IS LESS THAN SOME PRESELECTED
C     VALUE, SIZE.  THE TERM TESTED IS THE LARGEST
C     ELEMENT OF THE PHI MATRIX.
C
      W=T/(DEN*(Z+1.))
      DO 8 I=1,N
      DO 8 J=1,N
      A4(I,J)=W*A1(I,J)+A4(I,J)
    8 A2(I,J)=A4(I,J)
C
C        COMPUTATION OF DEL IS COMPLETED WHEN THE TEST
C     ON PHI IS SATISFIED EXCEPT FOR MULTIPLICATION BY B.
C
      IF(TERM-SIZE)9,9,2
```

```
 9 DO 10 I=1,ND
   DO 10 J=1,MD
10 DEL(I,J)=0.0
   DO 11 I=1,N
   DO 11 J=1,M
   DO 11 K=1,N
11 DEL(I,J)=DEL(I,J)+A2(I,K)*B(K,J)
   PRINT 13 , Z, SIZE,T
   CALL PRINTER(A,N,N,ND,ND,8H A       )
   CALL PRINTER(B,N,M,ND,MD,8H B       )
   CALL PRINTER(PHI,N,N,ND,ND,8H PHI     )
   CALL PRINTER (DEL,N,M,ND,MD,8H DEL     )
   PRINT 12
12 FORMAT(1H1)
13 FORMAT(1H1,2X,25HSUBROUTINE PHIDEL  NUM =  ,F3.0,
  14X,7HSIZE = ,E10.4,4X,4HT = ,E10.4)
   END
```

```
      SUBROUTINE PHIDEL1(A,B,H,N,M,ND,MD,T,PHISTAR,DELSTAR)
C
      DIMENSION A(ND,ND),B(ND,MD),PHISTAR(ND,ND),DELSTAR(ND,MD),
     1H(ND,ND),A1(10,10),A2(10,10),A3(10,10),PHI(10,10),
     2DEL(10,10),D(10,10)
C
C *** SUBROUTINES REQUIRED. RECIP  AND  PRINTER.
C
C        PHIDEL1 COMPUTES THE STATE TRANSITION MATRIX IN
C     CANONIC COMPANION MATRIX FORM, PHISTAR, AND THE
C     CORRESPONDING DELSTAR FOR A LINEAR SAMPLED-DATA
C     SYSTEM WITH SCALAR FORCING FUNCTION. GIVEN THE
C     LINEAR TIME INVARIANT DIFFERENTIAL EQUATION,
C      XDOT = AX + BU, THE PHI AND DEL FOR THE STATE
C     DIFFERENCE EQUATION, X(K+1) = PHI*X(K) + DEL*U(K),
C     ARE FOUND BY SERIES EXPANSION OF E**AT. A MATRIX
C     D IS FORMED FROM (H H*PHI ... H*PHI**(N-1))TRANSPOSE.
C     THEN PHISTAR = D*PHI*DINVERSE, DELSTAR = D*DEL.
C
C       REFERENCE, OPT. ESTIMATION, IDENTIFICATION, AND
C     CONTROL BY R.C.K.LEE
C
C     SYMBOLS USED.
C        XDOT       N*1 VECTOR
C        X          N*1 STATE VECTOR
C        A          N*N COEFFICIENT MATRIX
C        B          N*M COEFFICIENT MATRIX
C        U          M*1 INPUT VECTOR
C        K              SAMPLE NUMBER, K=1,2,...
C        T              FIXED TIME BETWEEN SAMPLES
C        H          N*N OBSERVATION MATRIX
C                       Z(K) = H*X(K)
C        D          N*N MATRIX DEFINED ABOVE
C        A1,2,3         TEMPORARY STORAGE
C
```

```
      SIZE=0.1E-20
      DO 1 I=1,N
      DO 1 J=1,N
      PHI(I,J)=0.
      PHI(I,I)=1.
      DEL(I,J)=0.
      DEL(I,I)=T
      A1(I,J)=A(I,J)*T
    1 D(I,J)=A1(I,J)
      DEN=1.0
      Z=0.
    2 Z=Z 1.
      DEN=DEN*Z
      TERM=0.
      DO 4 I=1,N
      DO 4 J=1,N
      A3(I,J)=A1(I,J)/DEN
      IF(TERM-ABSF(A3(I,J)))3,4,4
    3 TERM=ABSF(A3(I,J))
    4 CONTINUE
      DO 5 I=1,N
      DO 5 J=1,N
      PHI(I,J)=PHI(I,J)+A3(I,J)
    5 A3(I,J)=A3(I,J)*DEN
      CALL PRODPD1(D,A3,N,N,N,A1,10,10,1 ,10,10,10)
C
C     PHI = I +SUMMATION( (A*T)**N/N FACTORIAL )
C       WHERE N = 1 TO INFINITY
C
C        COMPUTATION OF PHI IS COMPLETE WHEN THE LAST
C     TERM OF THE SERIES IS LESS THAN SOME PRESELECTED
C     VALUE, SIZE.  THE TERM TESTED IS THE LARGEST
C     ELEMENT OF THE PHI MATRIX.
C
      W=T/(DEN*(Z+1.))
```

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```
      DO 6 I=1,N
      DO 6 J=1,N
      DEL(I,J)=W*A1(I,J)+DEL(I,J)
    6 A2(I,J)=DEL(I,J)
C
C        COMPUTATION OF DEL IS COMPLETED WHEN THE TEST
C     ON PHI IS SATISFIED EXCEPT FOR MULTIPLICATION BY B.
C
      IF(TERM-SIZE)7,7,2
    7 CALL PRODPD1(A2,B,N,N,M,DEL,10,10,ND,MD,10,10)
      PRINT 14 , Z, SIZE,T
C
C        NOW CONVERT PHI AND DEL TO CANONIC COMPANION
C     MATRIX FORM, PHISTAR AND DELSTAR.
C
      DO 8  J=1,N
      DO 8  I=1,N
      D(I,J)=0.
    8 A2(I,J)=0.
      DO 9 I=1,N
    9 A2(1,I)=H(1,I)
      DO 11 I=1,N
      DO 10 J=1,N
   10 D(I,J)=A2(1,J)
      CALL PRODPD1(A2,PHI,1,N,N,A1,10,10,10,10,10,10)
      DO 11 K=1,N
   11 A2(1,K)=A1(1,K)
      CALL PRODPD1(D,DEL,N,N,M,DELSTAR,1 ,10,10,10,ND,MD)
      CALL PRODPD1(D,PHI,N,N,N,A1,10,10,10,10,10,10)
      EP= .000001
      CALL RECIP(N,EP,D,A2,KER,10)
      CALL PRODPD1(A1,A2,N,N,N,PHISTAR,1 ,10,10,10,ND,ND)
      CALL PRINTER(A,N,N,ND,ND,8H A       )
      CALL PRINTER(B,N,M,ND,MD,8H B       )
      CALL PRINTER(PHI,N,N,10,10,8H PHI     )
```

```
      CALL PRINTER(DEL,N,M,10,10,8H DEL     )
      CALL PRINTER(H,N,N,ND,ND,8H H       )
      CALL PRINTER(PHISTAR,N,N,ND,ND,8H PHISTAR)
      LLL=N-1
      DO 12 JK=1,LLL
      DO 12 JL=1,N
      PHISTAR(JK,JL)=0.
   12 PHISTAR(JK,JK+1)=1.
      CALL PRINTER(PHISTAR,N,N,ND,ND,8H PHISTAR)
      CALL PRINTER(DELSTAR,N,M,ND,MD,8H DELSTAR)
      PRINT 13
   13 FORMAT(1H1)
   14 FORMAT(/,1H1,2X,26HSUBROUTINE PHIDEL1  NUM = ,F3.0,
     14X,7HSIZE = ,E10.4,4X,4HT = ,E10.4)
      END
      SUBROUTINE PRODPD1(A,B,N,M,L,C,N1,N2,N3,N4,N5,N6)
      DIMENSION A(N1,N2),B(N3,N4),C(N5,N6)
C       THIS S/R IS FOR PHIDEL1 ONLY.
      DO 1 I=1,N5
      DO 1 J=1,N6
    1 C(I,J)=0.
      DO 2 I=1,N
      DO 2 J=1,L
      DO 2 K=1,M
    2 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      END
```

```
      SUBROUTINE PSD(A,DELTAT,M,IPRINT,FREQ,X,TAU)                    00000
      DIMENSION A(100),X(100),FREQ(100),TAU(100),ITITLE(12)           0001
C                                                                       2
C     CALLING ARGUMENTS FOR S/R POWER SPECTRAL DENSITY (PSD)            0003
C       A = AUTOCORRELATION R(TAU) WITH A (1) AT TAU=0                  0004
C       DELTAT = TIME BETWEEN CORR. SAMPLES IN SECONDS                  0005
C       M= NUMBER OF CORR. SHIFTS FOR R (TAU), TAU=0,M                  0006
C       IPRINT = PRINT FLAG, IF IPRINT = 1, DATA IS PRINTED             0007
C       AND A GRAPH OF PSD VS FREQ IS DRAWN.                            0008
C       FREQ = CENTER FREQUENCYS OF DISCRETE BANDS ANALYZED (CPS)       0009
C       X = THE POWER SPECTRAL DENSITY AS A FUNCTION OF FREQ.           0010
C                                                                       11
C          REFERENCE..TUKEY AND BLACKMAN BOOK ON PSD                    0012
C                                                                       13
C                                                                       14
C     FIND X(1) --- PSD AT FREQ = 0.0                                   0015
C                                                                       16
      ASUM=0.0                                                          0017
      FM=M                                                              0018
      PI=4.0*ATANF(1.0)                                                 0019
      CS1=COSF(PI/FM)                                                   0020
      SN1=SINF(PI/FM)                                                   0021
      CSL=CS1                                                           0022
      SNL=SN1                                                           0023
      DO 1 L=2,M                                                        0024
      AZ=(1.0+CSL)                                                      0025
      ASUM=ASUM+AZ*A(L)                                                 0026
      CSL1=CSL*CS1-SNL*SN1                                              0027
      SNL1=SNL*CS1+CSL*SN1                                              0028
      CSL=CSL1                                                          0029
      SNL=SNL1                                                          0030
    1 CONTINUE                                                          0031
C                                                                       32
      X(1)=0.5*(ASUM+A(1))/FM                                           0033
C                                                                       34
```

```
C   FIND X(K)---PSD FOR K=2,M                                          0035
C                                                                        36
      CSK=CS1                                                          0037
      SNK=SN1                                                          0038
      MZ=M+1                                                           0039
      DO 6 K=2,MZ                                                      0040
      ASUM=0.0                                                         0041
      CSKL=CSK                                                         0042
      SNKL=SNK                                                         0043
      CSL=CS1                                                          0044
      SNL=SN1                                                          0045
      DO 2 L=2,M                                                       0046
      AZ=(1.0+CSL)*CSKL                                                0047
      ASUM=ASUM+AZ*A(L)                                                0048
      CSL1=CSL*CS1-SNL*SN1                                             0049
      SNL1=SNL*CS1+CSL*SN1                                             0050
      CSL=CSL1                                                         0051
      SNL=SNL1                                                         0052
      CSKL1=CSKL*CSK-SNKL*SNK                                          0053
      SNKL1=SNKL*CSK+CSKL*SNK                                          0054
      CSKL=CSKL1                                                       0055
      SNKL=SNKL1                                                       0056
    2 CONTINUE                                                         0057
      IF(K-MZ)3,4,4                                                    0058
    3 DZ=1.0/FM                                                        0059
      GO TO 5                                                          0060
    4 DZ= .5/FM                                                        0061
    5 X(K)=DZ*(ASUM+A(1))                                              0062
      CSK1=CSK*CS1-SNK*SN1                                             0063
      SNK1=SNK*CS1+CSK*SN1                                             0064
      CSK=CSK1                                                         0065
      SNK=SNK1                                                         0066
    6 CONTINUE                                                         0067
C                                                                        68
C   X(K)=X(FREQ)=PSD FROM FREQ.=0.0 TO FMAX. CPS                       0069
```

```
C      NOTE, X(K) IS REALLY COMPUTED AS ENERGY WITH                    0070
C      RESPECT TO UNIT CHANGE OF INDEX K, NOT CPS.                     0071
C      COMB = CPS PER INCREMENT (K+1) -K                               0072
C                                                                        73
       ASUM=0.0                                                        0074
       DO 7 K=2,M                                                      0075
       ASUM=ASUM+X(K)                                                  0076
     7 CONTINUE                                                        0077
       XENGY=0.5*(X(1)+2.0*ASUM+X(M+1))                                0078
C                                                                        79
C    FIND FRACTION OF TOTAL ENERGY IN CALCULATED FREQ. RANGE           0080
C      XFACT = 1.0 IF ALL FREQS. HAVE BEEN ACCOUNTED FOR               0081
C                                                                        82
       XFACT=XENGY/A(1)                                                0083
C                                                                        84
C    NOW OBTAIN PSD W.R.T. CPS BY NORMALIZING W.R.T. ENERGY            0085
C                                                                        86
       FREQ(1)=0.0                                                     0087
       TAU(1)=0.0                                                      0088
       AZ=2.0*DELTAT*FM                                                0089
       COMB=1.0/AZ                                                     0090
       DO 8 K=1,MZ                                                     0091
       X(K)=AZ*X(K)/XENGY                                              0092
       FREQ(K+1)=FREQ(K)+COMB                                          0093
       TAU(K+1)=TAU(K)+DELTAT                                          0094
     8 CONTINUE                                                        0095
C                                                                        96
C    POWER SPECTRUM PRINT-OUT INSTRUCTIONS                             0097
C                                                                        98
       PRINT 100,XFACT,A(1)                                            0099
       PRINT 101,COMB,FREQ(MZ)                                         0100
       IF(IPRINT-1)11,9,11                                             0101
     9 PRINT 102                                                       0102
       PRINT 103,(TAU(K),A(K),FREQ(K),X(K),K=1,MZ)                     0103
   100 FORMAT(8H XFACT =,F8.5,9H   A(0) =,E12.5,//)                    0104
```

```
101 FORMAT(8H COMB  =,F9.5,10H FMAX CPS=,F12.5,//)                          0105
102 FORMAT(53H   TAU(SEC)        R(TAU)       FREQ(CPS)   PSD(FREQ)    )   0106
103 FORMAT(/,1X,F11.7,3X,F10.5,3X,F9.3,3X,F10.5)                           0107
    CALL PTPLOT(FREQ,X,M)                                                  0120
    LABEL=4HPSD                                                            0108
    DO 10 I=1,12                                                           0109
10  ITITLE(I)=8H                                                           0110
    ITITLE(1)=8HMCDONOUG                                                   0111
    ITITLE(2)=8HH  0575                                                    0112
    ITITLE(4)=8HPOWER SP                                                   0113
    ITITLE(5)=8HECTRAL D                                                   0114
    ITITLE(6)=8HENSITY                                                     0115
    ITITLE(10)=8H  PSD(TA                                                  0116
    ITITLE(11)=8HU) VS. F                                                  0117
    ITITLE(12)=8HREQUENCY                                                  0118
 11 CONTINUE                                                               0121
    END                                                                     122
```

```
      SUBROUTINE   RECIP(N,EP,B,X,KER,M)                          00000
      DIMENSION B(M,M),X(M,M),A(10,10)                            00001
      DO 1 I=1,M
      DO 1 J=1,M
      A(I,J)=B(I,J)                                              000035
    1 X(I,J)=0.                                                   00004
      DO 2 K=1,N                                                  00005
    2 X(K,K)=1.                                                   00006
   10 DO 34 L=1,N                                                 00007
      KP=                                                            8
      Z=0.                                                        00009
      DO 12 K=L,N                                                 00010
      IF(Z-ABSF(A(K,L)))11,12,12                                   0011
   11 Z=ABSF(A(K,L))                                              0012
      KP=K                                                        00013
   12 CONTINUE                                                    00014
      IF(L-KP)13,20,20                                             0015
   13 DO 14 J=L,N                                                 00016
      Z=A(L,J)                                                    00017
      A(L,J)=A(KP,J)                                              00018
   14 A(KP,J)=Z                                                   00019
      DO 15 J=1,N                                                 00020
      Z=X(L,J)                                                    00021
      X(L,J)=X(KP,J)                                              00022
   15 X(KP,J)=Z                                                   00023
   20 IF(ABSF(A(L,L))-EP)50,50,30                                  0024
   30 IF(L-N)31,34,34                                              0025
   31 LP1=L+1                                                     00026
      DO 36 K=LP1,N                                               00027
      IF(A(K,L))32,36,32                                           0028
   32 RATIO=A(K,L)/A(L,L)                                         00029
      DO 33 J=LP1,N                                               00030
   33 A(K,J)=A(K,J)-RATIO*A(L,J)                                  00031
      DO 35 J=1,N                                                 00032
   35 X(K,J)=X(K,J)-RATIO*X(L,J)                                  00033
```

```
36 CONTINUE                                                    00034
34 CONTINUE                                                    00035
40 DO 43 I=1,N                                                 00036
   I1=N+1-I                                                    00037
   DO 43 J=1,N                                                 00038
   S=0.                                                        00039
   IF(I1-N)41,43,43                                             0040
41 IIP1=I1+1                                                   00041
   DO 42 K=IIP1,N                                              00042
42 S=S A(I1,K)*X(K,J)                                          00043
43 X(I1,J)=(X(I1,J)-S)/A(I1,I1)                                00044
   KER=1                                                       00045
   RETURN                                                      00046
50 KER=2                                                       00047
   PRINT 51
51 FORMAT(/,2X,15HSINGULAR MATRIX )
   END                                                           48
```

```
      SUBROUTINE ADD (A,B,N,M,C,ND,MD)                                  01
      DIMENSION A(ND,MD),B(ND,MD),C(ND,MD)                              02
      DO 152 I=1,N                                                      03
      DO 152 J=1,M                                                      04
  152 C(I,J) = A(I,J) + B(I,J)                                          05
      END                                                                6


      SUBROUTINE SUB (A,B,N,M,C,ND,MD)                                  07
      DIMENSION A(ND,MD),B(ND,MD),C(ND,MD)                              08
      DO 152 I=1,N                                                      09
      DO 152 J=1,M                                                      10
  152 C(I,J) = A(I,J) - B(I,J)                                          11
      END                                                               12


      SUBROUTINE TRANS(A,N,M,C,ND,MD)                                   21
      DIMENSION A(ND,MD),C(MD,ND)                                       22
      DO 153 I = 1,N                                                    23
      DO 153 J=1,M                                                      24
  153 C(J,I) = A(I,J)                                                   25
      END                                                               26
```

```
      SUBROUTINE PRINTER(OUT,N,M,ND,MD,INAME)
      DIMENSION OUT(ND,MD)
      PRINT 2,N,M,INAME
      DO 1 IR=1,N
    1 PRINT 3,(OUT(IR,IC),IC=1,M)
    2 FORMAT(/,10X,I2,2H *,I2,3X,A8)
    3 FORMAT(6X,10E13.4)
      END
```

```
      SUBROUTINE MATREAD(A,N,M,ND,MD,FIELD)
      DIMENSION A(ND,MD)
      DO 10 I=1,N
   10 READ FIELD,(A(I,J), J=1,M)
      END
```

```
      SUBROUTINE PROD (A,B,N,M,L,C,ND,MD,LD)                    13
      DIMENSION A(ND,MD),B(MD,LD),C(ND,LD)                      14
      DO 1 I=1,ND
      DO 1 J=1,LD
    1 C(I,J)=0.0
      DO 151 I=1,N                                              15
      DO 151 J=1,L                                              16
      DO 151 K = 1,M                                            18
  151 C(I,J) = C(I,J) + A(I,K)*B(K,J)                           19
      END                                                       20
```

```
      SUBROUTINE PTPLOT(X,Y,K)
      DIMENSION X(501),Y(500),IPLOTA(90),IPLOTB(120)
C
C        ARGUMENTS FOR PTPLOT
C
C  X= ABSCISSA, PRINTED VERTICALLY ON THE PRINTER
C  Y= ORDINATE,  PLOTTED ACROSS THE PAPER
C  K= NUMBER OF POINTS, 3 TO 500.
C
C  LET X BE AN INDEXING VARIABLE, EG. TIME, FOR BETTER OUTPUT
C
      X(K 1)=X(K)
      IF(K-2)2,2,1
    1 IF(K-500)3,3,2
    2 PRINT 100,K
  100 FORMAT(/,1X,11HIMPROPER K=,I5)
      RETURN
    3 DO 4 I=1,90
    4 IPLOTA(I)=1H
      DO 5 I=1,120
    5 IPLOTB(I)=1H.
      IPLOTC=1H+
      YMAX=Y(1)
      YMIN=Y(1)
      DO 9 I=1,K
      IF(Y(I)-YMAX)7,7,6
    6 YMAX=Y(I)
    7 IF(Y(I)-YMIN)8,9,9
    8 YMIN=Y(I)
    9 CONTINUE
      IF(YMIN - YMAX)61,60,61
   60 PRINT 106,YMIN
      RETURN
   61 CONTINUE
      PRINT 107
```

```
      PRINT 101,(IPLOTB(I),I=1,118)
  101 FORMAT(/,1X,118A1)
      PRINT 102,YMIN,YMAX
  102  FORMAT(1X,1H.,E10.5,70X,E10.5,1H.,5X,1HX,11X,1HY,7X,1H.)
      AY=ABSF((YMAX-YMIN)/89.0)
      DO 38 I=1,K
      IYPLOT=ABSF((YMIN-Y(I))/AY)
      IF((MODF(ABSF((YMIN-Y(I))/AY),AY)) AY/2.0)11,11,10
   10 IYPLOT=(IYPLOT+1)
      IF(IYPLOT-90)11,41,41
   41 IYPLOT=89
   11 IF(YMIN)12,12,29
   12 IF(YMAX)29,13,13
   13 IF(MODF(ABSF(YMIN/AY),AY)-AY/2.0)14,14,15
   14 IXAXIS=ABSF(YMIN/AY)
      GO TO 16
   15 IXAXIS=(ABSF(YMIN/AY)+1.0)
      IF(IXAXIS-90)16,40,40
   40 IXAXIS=89
   16 IF(X(I))52,17,50
   50 IF(X(I+1))21,26,26
   52 IF(I-1)19,19,53
   53 IF(X(I-1))19,19,28
   17 KK=IYPLOT+2
      PRINT 103,(IPLOTB(J),J=1,IYPLOT),IPLOTC,(IPLOTB(J),J=KK,
     190),X(I),Y(I)
  103  FORMAT(1X,1H.,90A1,1H.,1X,E10.5,2X,E10.5,2X,1H.)
      GO TO 38
   19 IF(I-K)20,22,22
   20 IF(X(I+1))22,22,21
   21 IF(ABSF(X(I))-ABSF(X(I+1)))17,17,22
   22 IF(IYPLOT-IXAXIS)23,24,25
   23 KK=IYPLOT+2
      LL=IXAXIS+2
      PRINT 103,(IPLOTA(J),J=1,IYPLOT),IPLOTC,(IPLOTA(J),J=KK,
```

```
     1IXAXIS),IPLOTB(1),(IPLOTA(J),J=LL,90),X(I),Y(I)
      GO TO 38
   24 L =IXAXIS+2
      PRINT 103,(IPLOTA(J),J=1,IXAXIS),IPLOTC,(IPLOTA(J),J=LL,
     190),X(I),Y(I)
      GO TO 38
   25 KK=IYPLOT+2
      LL=IXAXIS+2
      PRINT 103,(IPLOTA(J),J=1,IXAXIS),IPLOTB(1),(IPLOTA(J),J=LL,
     1    IYPLOT),IPLOTC,(IPLOTA(J),J=KK,90),X(I),Y(I)
      GO TO 38
   26 IF(I-1)22,22,27
   27 IF(X(I-1))28,22,22
   28 IF(ABSF(X(I-1))-ABSF(X(I)))22,22,17
   29 IF(X(I))54,30,51
   51 IF(X(I+1))33,35,35
   54 IF(I-1)31,31,55
   55 IF(X(I-1))31,31,37
   30 KK=IYPLOT+2
      PRINT 103,(IPLOTB(J),J=1,IYPLOT),IPLOTC,(IPLOTB(J),J=KK,
     190),X(I),Y(I)
      GO TO 38
   31 IF(I-K)32,34,34
   32 IF(X(I+1))34,34,33
   33 IF(ABSF(X(I))-ABSF(X(I+1)))30,30,34
   34 KK=IYPLOT+2
      PRINT 103,(IPLOTA(J),J=1,IYPLOT),IPLOTC,(IPLOTA(J),J=KK,
     190),X(I),Y(I)
      GO TO 38
   35 IF(I-1)34,34,36
   36 IF(X(I-1))37,34,34
   37 IF(ABSF(X(I-1))-ABSF(X(I)))34,34,3
   38 CONTINUE
      PRINT 104,(IPLOTB(J),J=1,118)
  104 FORMAT(1X,118A1)
      PRINT 105,AY
  105 FORMAT(3X, 9HDELTA Y =,F10.4)
  106 FORMAT(/,1X,17HY IS A CONSTANT =,F10.4,11H FOR ALL X.)
  107 FORMAT(1H1)
      RETURN
      END
```

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13. ABSTRACT

This thesis proposes a method of identifying the dynamics of ship angular motion at sea as the basis for ship motion estimation. Various sources of information are discussed. Identification algorithms are presented with the results of their stochastic digital simulation. Sensitivity of identification error to sample rate was investigated. A plan for shipboard implementation utilizing a digital computer is presented.

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